DOCUMENT RESUME

ED 431 615 SE 062 594

AUTHOR	Uy, Frederick Lim
TITLE	Geometry in the Middle Grades: A Multicultural Approach.
PUB DATE	1996-00-00
NOTE	254p.; Ed.D. Dissertation, Columbia University.
PUB TYPE	Dissertations/Theses - Doctoral Dissertations (041) -- Reports - Descriptive (141)
EDRS PRICE	MF01/PC11 Plus Postage.
DESCRIPTORS	Elementary Secondary Education; *Geometry; Mathematics Curriculum; *Mathematics Instruction; Middle School Students; Middle Schools; *Multicultural Education

ABSTRACT

After appropriate research, 18 geometry lessons were created using a multicultural approach. The lessons were designed to replace portions of a middle grades geometry curriculum dependent upon standard textbooks, and were piloted in an independent New York City school. The study involved 46 students and lasted for six weeks. The lessons were divided into four units. At the start of each, students were given a mathematical pre-assessment. After the entire unit had been taught, the students completed a post assessment on both the mathematical and the cultural topics. Additionally, students were asked to complete a questionnaire and were interviewed. A daily log of observations was maintained throughout the field trial. Finally, a five-member jury reviewed the lessons and completed an evaluation form. This study supported the claims that (1) students appreciate the contributions of cultures that are different from their own, and (2) linking the study of mathematics with other disciplines and cultures provides more meaning to the mathematics studied. When students were asked why they enjoyed the multicultural approach, most indicated that they saw uses and applications of mathematics outside the classroom and in other cultures that they had not encountered in previous mathematics classes. Also, students appeared to realize that certain mathematics topics could be connected to other disciplines. The results of this study suggest that many students appreciate mathematics topics because they see a direct and human way of applying them. The students appeared to be highly motivated and involved with the lessons, and classroom discussions were lively with broad participation. The jury indicated that (1) there was a nice flow of topics, (2) the sequencing of the lessons was adequate and moved from less difficult to more difficult, and (3) lessons were appropriate for middle grades. Jury members suggested that there should be more in-depth cultural and historical background for each lesson and agreed that the materials fostered awareness, appreciation, and acknowledgment of other cultures. (Contains 62 references.) (Author/NB)

ABSTRACT

GEOMETRY IN THE MIDDLE GRADES: A MULTICULTURAL APPROACH

Frederick Lim Uy

After appropriate research, the investigator created 18 geometry lessons using a multicultural approach. The lessons were designed to replace portions of a middle grades geometry curriculum dependent upon standard textbooks and were piloted in an independent New York City school. The study involved 46 students and lasted for six weeks.

The lessons were divided into four units; at the start of each, students were given a mathematical pre-assessment. . After the entire unit had been taught, the students completed a post-assessment on both the mathematical and the cultural topics. Additionally, students were asked to complete a questionnaire and were interviewed. The investigator maintained a daily log of his

observations throughout the field trial. Finally, a five-member jury reviewed the lessons and completed an evaluation form supplied by the investigator.

This study supported the claims that (1) students appreciate the contributions of cultures that are different from their own and (2) linking the study of mathematics with other disciplines and cultures provides more meaning to the mathematics studied. When students were asked why they enjoyed the multicultural approach, most indicated that they saw uses and applications of mathematics outside the classroom and in other cultures that they had not encountered in previous mathematics classes. Also, the students appeared to realize that certain mathematics topics could be connected to other disciplines.

The results of this study suggested that many students appreciated the mathematics topics because they saw a direct and human way of applying them. The students appeared to be highly motivated and involved with the lessons, and classroom discussions were lively with broad participation.

The jury indicated that (1) there was a nice flow of topics, (2) the sequencing of the lessons was adequate and moved from less difficult to more difficult, and (3) the lessons were appropriate for middle grades. The jury members suggested that there should be more in-depth cultural and historical background for each lesson and agreed that the materials fostered awareness, appreciation, and acknowledgment of other cultures.

GEOMETRY IN THE MIDDLE GRADES:

A MULTICULTURAL APPROACH

by

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Submitted in partial fulfillment of the
requirements for the Degree of Doctor of Education in
Teachers College, Columbia University
1996

ACKNOWLEDGMENTS

I express my sincerest gratitude to Professor J. Philip Smith, my advisor. It has been both a privilege and a pleasure to work under his expertise and supervision. His patience, encouragement, and constant advice had helped me accomplish a lot. Also, I thank Professors Bruce R. Vogeli and Eileen Donoghue for giving helpful suggestions and guidance.

I also thank Professor Ubiratan D'Ambrosio, Claudia Zaslavsky, Dr. Vera Preston, Marguerite Fredrick and Rim An. Without them, this study would not have become a reality.

My colleagues in United Nations International School also gave endless support. My special thanks go to them.

Lastly, I could not have completed this study without my family's support. I thank my parents, my sisters and brother, and all other members of my family. They never failed to believe in me and always gave me reassurance and inspiration. I also wish to thank my dearest friends; the encouragement and the cheers they gave me were overwhelming.

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Chapter I

INTRODUCTION

Need for the Study

After a thorough evaluation of the status of U.S. mathematics education across all grade levels, the National Research Council (NRC) in 1989 published Everybody Counts, which reported both the strengths and weaknesses of mathematics education in the country. According to the report, (1) one in every three American students will be minority by the year 2000 and by 2020, the minorities of today's population will become the majority of students in the United States; and (2) of those under 18 years of age, the proportion of minorities is already nearing 40 percent, almost three times what it was just after World War II (NRC,1989). Between the years 1979 and 1989, the total number of children ages 8 to 15 enrolled in U.S. schools who spoke a language other than English at home increased by 41 percent (Bruder, 1992).

Given the increasing multicultural make-up of the student population, it is sensible to reexamine our teaching approaches and to ponder the role of multiculturalism in our teaching and learning. Why would mathematics be a vehicle for multiculturalism? In Multicultural Mathematics Education for the Middle Grades, Zaslavsky (1991) wrote that "doing mathematics is a universal activity and people all over the world have developed mathematical practices consistent with their needs and interests" (p. 8). Also, in using a multicultural approach to mathematics, "the teacher is helping to overcome the existing deep-rooted Eurocentric bias relating to the origins and practices of mathematics" (Joseph, et al., 1993, p. 7). Furthermore, a multicultural approach to mathematics "helps to promote a 'holistic' view of learning, and provides an invaluable aid to an education in awareness" (Joseph, et al., 1993, p. 8). Mathematics that is taught with consideration for the cultural, racial, ethnic, and religious backgrounds of the students will encourage certain students who often exhibit little or no interest in the subject (Vogeli, 1993).

Given such recommendations, it is reasonable to review and revise the mathematics curriculum to accommodate the needs of students with diverse backgrounds. The National Council of Teachers of Mathematics in Curriculum and Evaluation Standards for School Mathematics recommends that "whenever possible, the cultural backgrounds of the students should be integrated into the learning experience" (NCTM, 1989, p. 68). The September, 1993 issue of the Middle School Journal, a publication of the National Middle School Association (NMSA), suggested that middle level school educators should "(1) provide culturally appropriate activities, and (2) provide teaching and learning experiences which reflect culturally diverse students' learning styles and the learner's role in the teaching and learning process" (Manning, 1993, pp. 16-17). These suggestions certainly reflect the significance of a multicultural approach to education and to mathematics. Implementing a multicultural approach in the classrooms can increase student awareness of different cultures. Also, a better understanding of multicultural education and its relevance to the middle grades mathematics curriculum will further enhance culture-based research in the field of mathematics.

Purpose of the Study

The purpose of this study is to develop a sequence of curricular lessons in geometry for the middle grades using a multicultural approach. A second purpose of this study is to subject the lessons to classroom trials with students from the middle grades.

This study will focus on the following questions:

1) What cultures are appropriate for inclusion in the middle grades geometry curriculum?
2) Which lessons appear to help students learn the required mathematical skills?
3) Which lessons appeal least (most) to the students?
4) What are the reactions of the students to the lessons?
5) What are the advantages and/or disadvantages in using a multicultural approach in the teaching of geometry?
6) Do the students report any appreciation for the different cultures after being taught using a multicultural approach?

Procedures of the Study

To accomplish the goals of this study, the investigator reviewed literature in both the geometry curriculum of the middle grades and in multicultural education. After surveying several middle grades mathematics textbook series, the lessons were identified and selected to be included in the study. Then, resources for multicultural education and multicultural mathematics were reviewed. The lesson plans for the chosen topics were then developed. (The lesson plan format was the one suggested by Posamentier and Stepelman (1990).) In addition to the lesson plans themselves, a list of related references was also created.

Two middle grades classes from an independent school in New York City were chosen to participate in the trial of the curricular materials. The investigator taught the selected classes using the developed materials for a period of about six weeks. At the start of each of four units, the participants were given a pre-assessment. The lessons in each unit were then taught by the investigator. After each unit was completed, the participants were given a post-assessment and were asked to complete a student questionnaire. At the conclusion of the four units, students were interviewed to substantiate their responses to the questionnaires. The students' performances on the assessments and their responses to the questionnaires were carefully analyzed.

After the classroom trials, the lessons were submitted to a five-member jury of mathematics educators-- three of whom are authorities on multicultural mathematics and ethnomathematics. The other two are middle grades mathematics teachers. They were asked to complete an evaluation form to critique the lesson plans and these evaluations were reported in a summary format. These evaluations would be used to revise the curricular materials and for recommendations for further study and research.

Plan of the Report

This report consists of five chapters and four appendices.

Chapter II reviews the literature on multicultural education and multicultural mathematics.

Chapter III describes how the lesson plan format was chosen, how the lessons were developed and how the pre-assessment, post-assessment, students questionnaire, and jury evaluation form were created. This chapter also describes the selection of the classes participating in the study, the administration of the pre- and post-assessments and student questionnaires, the teaching of the lessons, and the student interviews.

Chapter IV lists the results of the study and then discusses the analysis of the results, including the results of the completed student questionnaires and of the interviews, the performances of the students in both the pre- and post-assessments and the summary of the jury's evaluation of the lessons.

Chapter V gives a summary of the study, lists the conclusions of the study, and presents revisions of the lesson plans based on the evaluations and recommendations of the five-member jury. Additionally, the last chapter includes recommendations for further research and study in using a multicultural approach in middle grades geometry.

Chapter II

BACKGROUND FOR A MULTICULTURAL EDUCATIONAL STUDY

Synopsis of the Study

After surveying the literature on multicultural education and some middle grades mathematics textbook series, the investigator wrote 18 geometry lessons using a multicultural approach. These lessons were designed to replace portions of a middle grades geometry curriculum dependent upon standard textbooks and were piloted in an independent New York City school. The study lasted for six weeks and 46 students participated in the study.

The lessons were divided into four units and, at the start of each unit, the students were given a pre-assessment on the mathematical topics. After the entire unit had been taught, the students completed a post-assessment on both the mathematical and the cultural topics. Additionally, they were asked to complete a

questionnaire and were interviewed. The investigator maintained a daily log of his observations throughout the field trial. Finally, a five-member jury reviewed the lessons and completed an evaluation form supplied by the investigator.

An Educational Concern

The launching of Sputnik by the Soviets in 1957 triggered an outpouring of concern about the quality of education in the United States. Educators came to feel that curriculum reform was necessary if the United States was to maintain its preeminent international role. The 1960's, 1970's and 1980's saw the birth of many curricular innovations: site-based management, cooperative learning, cross-age groupings, writing across the curriculum, and multicultural education, to name a few (Bruder, 1992). The application of a multicultural approach to education is today a growing field for curricular innovators and researchers. To understand a multicultural approach to education, one must look into its history, its current status, and its future.

The History of Multicultural Education

Multicultural education is a product of several education movements that started in the 1960's. When black residents of the Watts District in Los Angeles violently demonstrated their wrath and discontent, the belligerent black protest movement was born (Banks, 1993). Banks suggested that the black protest movement spawned educational programs like black studies, ethnic studies and multiethnic education, which eventually gave rise to multicultural education (Banks, 1993). Likewise, Baker notes,

> In the early sixties, African Americans, Latinos, Native Americans, and Asian and Pacific Islanders were demanding that greater efforts be made toward equality. The Civil Rights Act of 1964 was a triumph, giving support to the movement that public education reflect greater sensitivity to the needs and values of these and other ethnic groups. (Baker, 1994, p. 17)

The concern for multiethnic education continued through the 1970's. By that time, the study of the different ethnic groups in the United States was fast becoming a part of the curriculum. However, an issue regarding how ethnicity was perceived surfaced (Baker, 1994). Marden and Meyer (as cited in Baker, 1994) defined ethnicity as "a term which emphasizes the cultural ethos (values, expectations, symbols) of a group and formerly, quite properly, was limited in reference to groups whose cultural characteristics are their prime distinguishing factors" (Baker, 1994, p. 18). In contrast, Banks (as cited in Baker, 1994) believed that ethnicity was established by a group of people who "share a sense of group identification, a common set of values, political and economic interests, behavior patterns and other cultural elements which differ from those of other groups within a society" (Baker, 1994, p. 18). An analysis of these definitions revealed that Marden and Meyer believed that ethnicity was established by outside influences while Banks believed that ethnicity was already present in any society, established by internal influences. Multiethnic education was further complicated by the fact that multiethnic curricula tended to involve only individual and detached courses and targeted specific groups only (Baker, 1994, p. 18). Baker believed that multicultural education must be "comprehensive in nature" (Baker, 1994, p. 19). Since multiethnic education failed this requirement, multicultural education was conceived. Hence, multiethnic education led the way for the foundation of multicultural education

(Baker, 1994).

In Nieto's view, multicultural education was conceived as an answer to the "inequality in education based on racism, ethnocentrism, and language discrimination" (Nieto, 1992, p. xxviii). The American Association for Colleges and Teacher Education (AACTE) wrote, in 1972,

> Multicultural education rejects the view that schools should seek to melt away cultural differences or the view that schools should merely tolerate cultural pluralism. Instead, multicultural education affirms that schools should be oriented toward the cultural enrichment of all children and youth through programs rooted to the preservation and extension of cultural diversity as a fact of life in American Society, and it affirms that this cultural diversity is a valuable resource that should be preserved and extended. (AACTE, 1972)

Definitions of Multicultural Education

But what exactly is multicultural education? A sociopolitical definition was offered by Nieto:

> Multicultural education is a process of comprehensive school reform and basic education for all students. It challenges and rejects racism and other forms of discrimination in schools and society and accepts and affirms

> the pluralism (ethnic, racial, linguistic, religious, economic, and gender, among others) that students, their communities, and teachers present. Multicultural education permeates the curriculum and instructional strategies used in schools, as well as the interactions among teachers, students and parents, and the very way that schools conceptualize the nature of teaching and learning. Because it uses critical pedagogy as its underlying philosophy and focuses on knowledge, reflection, and action (praxis) as the basis for social change, multicultural education furthers the democratic principles of social justice. (Nieto, 1992, p. 208)

Davidman and Davidman (1994) saw at least three types of definitions of multicultural education: "those involving cultural pluralism, those involving equity, and those involving the lessening of racism, sexism and other -isms" (Davidman and Davidman, 1994, p. 19). Let us follow the Davidmans' classification and look at several definitions of multicultural education. The Association for Supervision and Curriculum Development (ASCD) in 1977 defined multicultural education as a "humanistic concept based on the strength of diversity, human rights, social justice, and alternative life choices for all people" (ASCD, 1977, p. 2). A similar definition was given as "that which recognizes and respects the cultural pluralistic nature of our society" (Baptiste and Baptiste, 1979, p. 44). Both these definitions refered to the culturally diverse make-up of the community in which we lived. However, alternative definitions based on equality also existed. Banks called multicultural education a movement dealing with fairness for different cultural and ethnic groups (Banks, 1981). Gollnick and Chinn wrote the following equity-based definition:

> Multicultural education is the educational strategy in which the student's cultural background is viewed as positive and essential in developing classroom instruction and a desirable school environment. It is designed to support and extend the concepts of culture, cultural pluralism, and equity into the formal school setting. (Gollnick and Chinn, 1986, p. 5)

The third set of definitions entailed those involving racism, sexism and other -isms. One such definition, taken from the Office of the Chancellor of the California State University System (1983), stated that multicultural education was viewed as a "methodology to encounter racism and prejudice based on ethnic identification and to promote positive attitudes about human diversity" (pp. 85 - 86). From his survey, Kim (as cited in Davidman and Davidman), in 1987, gave this definition of multicultural education:

> Multicultural education is a deliberate educational attempt to help students understand facts, generalizations, attitudes, and behaviors derived from their own ethnic roots (origins) as well as others. In this educational process students will unlearn racism (ethnocentrism) and recognize

> the interdependent fabric of our human society, giving acknowledgment for contributions made by various ethnic groups throughout the world. (Davidman and Davidman, 1994, p.21)

Justifications for Multicultural Education

In light of the history and definitions reviewed above, we now examine some of the justifications offered by the proponents for implementing a multicultural approach to education.

> Our nation's culturally diverse and pluralistic nature makes it imperative for schools to provide educational experiences and training that will not only prepare students to live successfully in a diverse nation but also to base educational content and process on the cultural histories, experiences, language, and lifestyles of all students. (Baker, 1994, p. 5)

Furthermore, Baker argues that even if cultural factors are disregarded, the curricula of the schools of today hardly meet the needs of our current students (Baker, 1994). If this is the case, then we must find a means to remedy the situation. Multicultural education offers a new method of reforming our schools since "it responds to many of the problematic factors leading to school underachievement and failure" (Nieto, 1992, p. 222). Finally, multicultural education calls for a defiance of the dominant powers that have long undermined some groups and blamed these groups' educational downfalls on their innate inadequacies (Cummins, 1991, pp. xviii - xix). These typical justifications explain the rationale behind a multicultural approach to education.

What then are the goals of multicultural education? Baker writes,

> 1) One goal of multicultural education is to help students become more aware of themselves as individuals and of their culture and/or cultures; 2) A second goal of multicultural education is to help individuals develop and understanding and appreciation for the cultures of others; 3) A third goal of multicultural education is to encourage individuals to support and to participate in as many different cultural groups as they choose; and 4) A fourth goal is to help individuals reach their full potential so that they are in control of their lives and thereby become empowered. (Baker, 1994, pp. 25 - 26)

On the other hand, the Davidmans offer six goals of multicultural education: "educational equity; empowerment of students and their parents; cultural pluralism in society; intercultural/interethnic/intergroup understanding and harmony in the classroom, school, and community; an expanded knowledge of various cultural and ethnic groups; and the development of students, parents, and practitioners whose thoughts and actions are guided by an informed and inquisitive multicultural perspective" (Davidman and Davidman, 1994, p. 2). A detailed comparison of the two sets of goals demonstrates that there are similarities and that some of these goals are overlapping. Baker's goals 1, 2, 3, and 4 are, respectively, similar to the Davidmans' cultural pluralism in society; intercultural/interethnic/intergroup understanding and harmony in the classroom, school, and community; an expanded knowledge of various cultural and ethnic groups; and empowerment of students and their parents. The remaining two goals of the Davidmans are direct outcomes of their first four goals.

Some Ongoing Controversies Surrounding Multicultural Education

Experts construe many different definitions of, and goals for, multicultural education. Partly for these reasons, it is not surprising to find controversies associated with the multicultural education movement. One controversy revolves around the claim of some experts that multicultural education "was and is a reform movement" (Davidman and Davidman, 1994, p. 24). The Davidmans assert that multicultural education has always been considered to be a revisionist approach to history and has served as a threat to the individuals who are satisfied with the way things are and who claim that a multicultural approach to education is just a reform movement and just like any other reform movement, it will be shortlived and soon be replaced by another. However, Nieto believes that such a fate is unlikely because multicultural education is "ongoing and dynamic and no one stops becoming a multicultural person; and knowledge is never complete" (Nieto, 1992, p. 218).

A second controversy is based on the alleged contribution of multicultural education to racial tensions. Schlesinger, an avid opponent of multicultural education, claims that multicultural education "glorifies ethnic and racial communities at the expense of common culture and glorifies ethnic and racial myths at the expense of honest history" and "promotes fragmentation, segregation and ghettoization" during this time of ethnic conflict that destroys one nation after another (Schlesinger, 1994, p. 17). Glazer disputes this allegation by stating that multicultural education aims to include underrepresented and unrepresented groups into the mainstream and relieve racial tensions (Glazer, 1994).

Still another controversy involves the renewed definition of "being an American" brought about by multicultural education (Davidman and Davidman, 1994, p. 24). The renewed definition of "being an American" nowadays does not refer only to European Americans; it also includes all those people whose ancestry is not European (Nieto, 1992, p. 271). Some people offer resistance to this definition and do not readily embrace this thought. Additionally, Richard-Amato and Snow acknowledged that the Anglo-American culture could no longer solely claim "the embodiment of all things 'American' ." Today's society is becoming more and more pluralistic (Richard-Amato and Snow, 1992, p. 1).

Finally, one more controversy regarding multicultural education arises out of the fear that a multicultural approach to education will divide the nation (Banks, 1993). Schlesinger claims that multicultural education "promotes fragmentation, segregation and ghettoization" (Schlesinger, 1994, p. 17). Banks disputes this claim, saying that anyone believing in this scenario definitely is assuming that the nation is already united; and this belief is not the case. He argues that while the nation is one politically, "sociologically the nation is deeply divided alone lines of race, gender, and class" (Banks, 1993, p. 23).

The Future of Multicultural Education

What does the future hold for multicultural education? Multicultural education will continue to be "implemented widely in the nation's schools, colleges and universities" (Banks, 1993, p. 27). The curriculum, the pedagogy, and other teaching-related disciplines will certainly reflect it. Green and Perlman certainly agree that the presence of multiculturalism in education will be felt more and more. They claim that educators are the secondary purveyors of culture, and, therefore, "must develop curricula and pedagogies that incorporate an understanding of cultural processes and cultural continuity and change within the framework of cultural diversity and American pluralism" (Green and Perlman, 1995, p. 6). Also, the continuing controversies about multicultural education will stay. As James Banks wrote, "These debates are consistent with the philosophy of a field that values democracy and diversity" (Banks, 1993, p. 27).

Multicultural Mathematics

If multicultural education will be implemented in our schools and our curriculum, it is only reasonable that this approach be spread throughout different disciplines, including mathematics. However, is mathematics really a sound vehicle for multiculturalism?

According to Joseph, "there is a view prevalent among mathematics teachers that the universal character of the language and reasoning of mathematics is sufficient evidence of its lack of cultural specificity" (Joseph, 1993, p. 6). This view is further strengthened when Banks recalls a mathematics teacher remarking that multicultural education is "appropriate for language arts and social studies teachers, ... math is math, regardless of the color of the kids" (Banks, 1993, p. 25). If this idea gains credence among the nation's teachers, then a multicultural education approach to mathematics will certainly be difficult. However, the belief that "math is math" is inaccurate. Historically, mathematics is believed to have developed from many different cultures (D'Ambrosio and

D'Ambrosio, 1994). This same mathematics presently is "taught and practiced at all levels all over the world, disregarding any cultural boundaries" (D'Ambrosio and D'Ambrosio, 1994, p. 689). Furthermore, Zaslavsky wrote: "Mathematical practices and concepts arose out of the real needs and interests of people in all societies, in all parts of the world, in all eras of time. All peoples have invented mathematical ideas to deal with such activities as counting, measuring, locating, designing, and yes, playing, with corresponding vocabulary and symbols to communicates their ideas to others." (Zaslavsky, 1993, p. 46). Finally, mathematics is an outcome of "human creation and decision-making, and connected with other realms of knowledge, culture and social life" (Ernest, 1991, p. 207). Through the views of experts like the foregoing, one sees that mathematics is indeed multicultural.

The Rationale for Multicultural Mathematics Education

Many experts agree that a multicultural approach should be used in the teaching of mathematics. A mathematics curriculum that does not involve culture, according to Secada, is likely to consider mathematics as knowledge belonging to some exclusive group (Secada, 1991, p. 49). Secada further argues that an impartial mathematics education needs to incorporate "real contexts that reflect the lived realities of people who are members of equity groups" (Secada, 1991, p. 49).

Nelson stated that a multicultural approach "enables one to develop the child's grasp of the extent of mathematics" and that this approach "can lead to a better appreciation of the history of ideas and the evolution of the subject" (Nelson, et al., 1993, p. 32).

Zaslavsky indicates that "mathematics is not just a white invention and a multicultural approach can help students of many different backgrounds take pride in the accomplishments of their people, whereas the failure to include such contributions in the curriculum implies that they do not exist" (Zaslavsky, 1991, p. 13). She further discusses the value of introducing multicultural and interdisciplinary approaches into the mathematics curriculum.

> (1) Students become aware of the role of mathematics in all societies. They realize that mathematical practices arose out of a people's needs and interests. (2) Students learn to appreciate the contributions of cultures different from their own, and to take pride in their own language. (3) By linking the study of mathematics with history, language arts,

> fine arts and other subjects, all the disciplines take on more meaning. (4) The infusions into the curriculum of the cultural heritage of people of color builds their self-esteem and encourages them to become more interested in mathematics. (Zaslavsky, 1990, p. 4)

In Diversity, Reform, and Professional Knowledge: The Need for Multicultural Clarity, Tate (1994) wrote that a multicultural approach "represents an effort to use mathematics to promote equity and fairness in the democracy" and "is consistent with contemporary visions of mathematics education" (p. 64). He supports his claims by asserting that this approach gives our students ample preparation to strive in the economy and the democracy and that this same approach teaches the students to think discriminately about mathematics and its purpose in society.

The mathematicians and researchers who convened in Budapest in 1988 for the 6th International Congress on Mathematical Education presented many papers, reports, and lectures that focused on the theme, "Mathematics, Education, and Society." Many presenters spoke and wrote of multicultural education and its connection to mathematics.

Dawe stated that "Throughout the world cultural diversity is commonplace in mathematics education. It has a profound influence on learning and teaching. Such a rich diversity between cultures, and therefore within mathematics as a whole, can considerably enrich the quality of mathematical activities in different classrooms around the world" (Dawe, 1989, p. 11). This argument gives a clear rationale for a multicultural approach in the teaching of mathematics.

Another justification for the multicultural approach in teaching mathematics is that teaching can be more effective and offer more equal opportunities, if it "starts from and feeds on the cultural knowledge or cognitive background of the subjects, and will differ in contents and strategy depending on the cultural background of the students" (Pinxten, 1989, p. 28).

In his paper, "Let Them Eat Cake" Desire, Cognition, and Culture in Mathematics Learning, Taylor (1989) asserted three goals for multicultural mathematics -- 1) it regards cultural forces as consistently deterministic, 2) it furnishes a starting point for the "compensatory education of culturally deprived students," and 3) it creates an education that is appropriate to the students' background and experiences (p. 162).

A similar array of rationales is given by Presmeg. She outlined four important purposes for using a multicultural approach in the teaching of mathematics.

> 1) Children need the stability of their cultural heritage, especially during periods of rapid social change. 2) The mathematics curriculum should incorporate elements of the cultural histories of all the people of the region. 3) The mathematics curriculum should be experienced as "real" by all children, and should resonate, as far as possible, with diverse home cultures. 4) The mathematics curriculum should be seen by pupils as relevant to their future lives. (Presmeg, 1989, p. 172)

Finally, there are those who believe that in using a multicultural approach to mathematics, "the teacher is helping to overcome the existing deep-rooted Eurocentric bias relating to the origins and practices of mathematics" (Joseph, et al. , 1993, p. 7). A multicultural approach to mathematics "helps to promote a 'holistic' view of learning, and provides an invaluable aid to an education in awareness" (Joseph, et al., 1993, p. 8).

Approaches to Multicultural Mathematics Education

Given that one wishes to use a multicultural approach to teach mathematics, how and where should one start? The most frequent suggestion: start with the curriculum. Nieto contends that most of the curriculum in the nation is biased toward European and American perspectives, which eliminates the viewpoints and positions of many students, and that this bias is demonstrated in the textbooks available (Nieto, 1992). Zaslavsky points out that open-ended projects are needed in the mathematics curriculum. These same projects must involve all students and must utilize their talent and abilities (Zaslavsky, 1991). The inclusion of the historical backgrounds of different mathematical topics is recommended by Nelson, who notes that this inclusion is supported by others, who see a need for the growth and extensive circulation of a true historical perspective (Nelson, 1993). Nelson also reminds mathematics educators of the necessity of seeking worthwhile mathematical tasks.

> Our task then is to seek out, on the one hand, cultural materials which offer mathematical gains, and, on the other, mathematical activities which make an effective contribution to multicultural education. (Nelson, 1993, p. 41)

Problems of a Multicultural Approach to Mathematics

Experts have identified some negative aspects associated with multicultural approaches to teaching mathematics.

Zaslavsky noted the inadequacy of curricular materials and insufficient training for teachers (Zaslavsky, 1989). These problems imply that more materials of the right kind be produced and that mathematics educators of today need more preparation in the area of multicultural curricula.

Nelson mentions three basic concerns accompanying multicultural approaches: the problem of inaccuracy and insecurity; the misuse of the approach; and the consideration of the approach as "irrelevant antiquarianism and, more seriously, as misleading" (Nelson, 1993, p. 39). Nelson notes that although most mathematics teachers can navigate their way out when they make errors with technical problems in mathematics, historical and cultural questions are not necessarily their area of expertise. Teachers may feel inadequate and insecure and argue that multiculturalism serves no useful purpose. Secondly, some teachers might opt to concentrate so much on unfamiliar cultures that important mathematical topics get neglected. Nelson gave a hypothetical example of a student introduced to different numerical systems from different cultures without any increase in the student's knowledge of decimal system. Finally, according to Nelson, some educators are skeptical about applying a multicultural approach, arguing that with advancement in technology, the children of today must be taught to move in accordance with this advancement. Rather than spending the time teaching multiculturalism, the instructional time is better used for some other pertinent areas like computers and telecommunications.

Studies Involving Multicultural Mathematics Education

Dawe claims that cultural diversity offers a way to enrich the quality of activities in mathematics teaching (Dawe, 1989). Studies of multicultural education tend to agree with this claim. Vogeli's (1992) study of the ethnomathematics of Southern Africa included development of five mathematics lessons intended for middle grades students categorized as Numeration, Relation, Geometry, Time and Space Measurement, and Games. An example of the lessons he developed was Sand Tracings in the Kalahari. The sand tracings, called sona, were drawn by connecting, with a continuous path, dots made in the sand with either a finger or a stick. This lesson belonged to the branch of mathematics called Graph Theory. Vogeli suggested that this lesson could serve as an extension of geometry lessons because it "reinforces the concepts of vertex, concurrence, and intersection as well as generalizing the concepts of adjacency and curve" (Vogeli, 1992, p. 160).

Although his lessons were not subjected to a formal classroom trial, these lessons were evaluated by a jury of mathematics and anthropology educators dealing with multicultural education. His study suggested that a multicultural approach to teaching mathematics can " 'personalize' and 'naturalize' the otherwise abstract mental exercises of mathematical topics that are seldom related to everyday activities and considerations of the student" (Vogeli, 1992, p. 101). This conclusion suggests that the students get to appreciate what they are learning since they now can see how what they learned is applied to real life.

Nobre (1989) utilized "Animal Lottery" in teaching probability to students in Brazil. Animal lottery is a popular game and played all over the country. In this game, 25 animals are each assigned a group number. Additionally, each of the animals is assigned four *tens* . For example, Animal A is assigned 01, 02, 03 and 04. Animal B is assigned 05, 06, 07 and 08. The last animal is assigned 97, 98, 99 and 00. Five prizes are then drawn. For example, if the first prize is 9034, one must look into the tens. 34 is part of the array 33, 34, 35 and 36. Hence, the first prize is the animal that is assigned this array of tens. The first two digits are used for betting and pay-off purposes. Nobre notes,

> "Animal Lottery" is admittedly the most popular of all games of chance practiced in Brazil and, among its followers, there are people from the several cultural and economic classes existing in the country. Most of its gamblers (which corresponds, in terms of percentage, to the majority of the population) are analphabet or semi-analphabet. Nevertheless, they make their bets, choose the most convenient kind of game, check the results and figure out how much they won (if so) very easily. They do calculations which appear to be enviable to many people who have spent years in school, but are not able to do them. (Nobre, 1989, p. 176)

According to Nobre, his students were very much motivated and the whole class was participating in the activity. Furthermore, he remarked that as "mathematics itself was begun to be seen as an accessible and easy-to-learn subject," his students became more motivated to study, and appeared to have a better understanding of the society to which they belonged (Nobre, 1989, p. 177).

A study by Pinxten involved spatial concepts and everyday mathematics of the Navajo Indians. He also conducted a similar study among the members of the Turkish immigrant community in Ghent, Belgium. In his paper World View and Mathematics Teaching, Pinxten discovered that Navajo students in a Western mathematics curriculum were not as successful as students in a bilingual program (where Navajo was the first language and English was the second). He reasoned that "the foundation for understanding and hence for translation was missing" (Pinxten, 1989, p. 29). According to Pinxten, "the part-whole distinction is absent in the Navajo knowledge system" and the "notions of horizontality, distance, etc. that Navajos use are constituted in an altogether different and incomparable way from the Western ones" (Pinxten, 1989, p.28). On the other hand, the Turkish immigrant groups had better success since their culture was more similar to the Western culture in which they lived. As a conclusion, Pinxten suggests,

> Let us study and develop the insights that are there in the pre-school knowledge system. The approach is very similar to 'English as a second language': To get to a rich semantics and a fluent use of language, it is important to

> develop and train the mother tongue first, and -- on the basis of the skills and insights with regard to the mother tongue -- then to eventually teach a second language. The same strategy should apply with regard to mathematics: First study, train, and develop the semantically rich notions about space, form, number, measure, etc. which are there. Study and train them in the language of the people and focus as much as possible on insightful learning of the knowledge which is there in the pre-school situation. (Pinxten, 1989, p. 29)

Zaslavsky has done many studies that involved the use of a multicultural approach in teaching mathematics. In one study, she discussed probability by tossing objects other than coins and dice. The people she observed made use of "cowrie shells, half-shells of nuts, and other objects appropriate to various societies" (Zaslavsky, 1989, p. 14). She noted that the students joined in the activities with a great deal of enthusiasm, due to the students' familiarity the objects.

Another lesson she discussed involved finding the shapes and the areas of different houses having the same perimeter. In this activity, the students were asked to sketch the shapes of the houses on a grid and count the squares inside each shape. As a result of this activity, Zaslavsky reported that one class "designed, constructed, and decorated several African-style compounds of round houses with conical roofs" (Zaslavsky, 1989, p. 15).

Finally, Langdon developed mathematics lessons for students in Ghana. In doing a lesson on symmetry and rotation, he used the designs of Adinkra cloth. The particular Adinkra design that was used was called "Nkonsonkonso" (Langdon, 1989). Langdon concluded that students learning mathematics have a more improved perception of mathematics by knowing that mathematics is found and is used in their own surroundings and that the students have discovered greater interest and accessibility in mathematics (Langdon, 1989).

Sources

Zaslavsky (1989) noted the inadequacy of curricular materials for a multicultural approach in teaching mathematics. Even though some curricular materials are available, not one is designed exclusively for middle grades geometry. Also, the available multicultural mathematics books oftentimes only write about the usefulness of a multicultural approach and how the lessons should be designed; however, very few examples on actual multicultural lessons are given.

A good start is Zaslavsky's Africa Counts (1973). This book gives an account of the mathematics present in some African cultures. This book discusses numeration and its uses, mathematical recreation and geometric patterns and shapes. Another valuable source is Ascher's Ethnomathematics (1991), that deals with the mathematics of some cultures devoid of Western influences. Topics for this book include numeration, sand tracings, kin relations, chance and strategy in games and puzzles, space modeling, and symmetry of different patterns. Non-African cultures like Native American, Maori, South Pacific, etc. are mentioned. A third book, The Crest of the Peacock, Non-European Roots of Mathematics (Joseph, 1991) discusses the history and contributions of the Egyptians, Chinese, Indians and Arabs to the present-day mathematics. Multicultural Mathematics by Nelson, Joseph and Williams (1993) offers suggestions on teaching mathematics from a multicultural perspective. These four books detail the significance of a multicultural perspective in teaching mathematics; however, the given mathematics lessons using a multicultural approach in these sources are few.

Krause's Multicultural Mathematics Materials (1983) gives different mathematical activities from various parts of the world. This book suggests multicultural activities but does not mention for which particular mathematical topics they are intended. In 1984, the Seattle Public Schools developed lessons on multicultural mathematics. The learning packet contains posters for the activities and the suggested activities are for different mathematical topics and not limited to geometry. Further, this compilation of lessons is intended for the upper middle grades and for high school students. As mentioned earlier, Vogeli (1992) developed mathematics lessons that were meant for middle grades students and were based on the ethnomathematics of Southern Africa. Although his lessons are a rich source for a multicultural approach in teaching middle grades mathematics, there is only one lesson that is connected to geometry (Sand Tracings in the Kalahari). Zaslavsky (1993) then wrote Multicultural Mathematics whose lessons were categorized into Numbers, Geometry and Measurement, Probability, Statistics and Graphs, and Fun with Math. This book grouped geometry lessons in one unit. These activities involved not only a multicultural approach, but also interdisciplinary and cooperative learning approaches. Addison-Wesley worked with many writers to develop Multiculturalism in Mathematics, Science and Technology: Reading and Activities. Together with the book is a representation of the world, Wall Chart, that serves as a visual aid to both the teachers and the students. This chart shows from which part of the world the lessons originate. Although many of the activities are intended for high school students, some of them can be adapted to middle school students. Lastly, Irons and Burnett (1994) wrote their series Mathematics From Many Cultures, which is intended for K - 5 students. This series provides big, colorful posters for discussions and worksheets for the activities. Accompanying each grade level is a separate book called Teachers' Notes where cultural and historical background materials and blackline masters are included. Although the audience for this series is primarily junior school students, some of the activities are easily adjusted to the needs of the middle school.

Some publishers have now included multicultural sections in their new textbook series. A good example is Mathematics Plus (1994) by Harcourt Brace and Company. In each chapter, a section called Multicultural Connection appears and this introduces both the teacher and the students to some multicultural application of a particular mathematical topic. Other publishers, like Silver Burdett & Ginn and Addison-Wesley, have augmented their textbook series by publishing ancillary materials that deal with multiculturalism. In each grade level, both publishers developed learning packets that suggest multicultural connections associated with the mathematical topics in that specific grade level.

In summary, this chapter has discussed the origins/history, definitions, goals, controversies and the future of multicultural education and the rationale, the approach, and the problems for its implementation in the teaching of mathematics and in the mathematics curriculum. Zaslavsky notes,

> Bringing the world into the mathematics class by introducing both cultural applications and current societal issues does motivate and empower students. Such mathematical content offers wonderful opportunities for project work, cooperative learning, connections with other subject areas, and community involvement. To carry out such a program effectively requires a new approach to curriculum development, teacher education, and assessment processes. (Zaslavsky, 1993, p.54)

Chapter III

THE CREATION AND TRIAL OF THE CURRICULAR MATERIALS

Synopsis of the Study

After surveying the literature on multicultural education and some middle grades mathematics textbook series, the investigator wrote 18 geometry lessons using a multicultural approach. These lessons were designed to replace portions of a middle grades geometry curriculum dependent upon standard textbooks and were piloted in an independent New York City school. The study lasted for six weeks and 46 students participated in the study.

The lessons were divided into four units and, at the start of each unit, the students were given a pre-assessment on the mathematical topics. After the entire unit had been taught, the students completed a post-assessment on both the mathematical and the cultural topics. Additionally, they were asked to complete a

questionnaire and were interviewed. The investigator maintained a daily log of his observations throughout the field trial. Finally, a five-member jury reviewed the lessons and completed an evaluation form supplied by the investigator.

The Process

In order to select topics appropriate for this study, the investigator, who has considerable teaching experience in the middle grades, examined different series of middle grades mathematics textbooks and consulted the New York State Education Department's Mathematics Syllabus. Topics appropriate for multicultural applications were sought out particularly. Finally, a balanced selection of topics suitable for the middle grades was chosen for the study. The topics resulted in 18 lessons and were grouped into four units. The reasons for grouping the topics were two-fold: 1) to arrange related topics together, and 2) to make the study more manageable.

The first unit, dealing with geometric concepts, contains the following seven lessons: Angles, Parallel & Perpendicular Lines, Transversals, Triangles, Quadrilaterals, Polygons, and Circles. The second unit, dealing with geometric measurements, has three lessons: Right Triangles, Perimeter & Area of Polygons, and Circumference & Area of Circles. The third unit, dealing with geometric transformations, has four lessons: Symmetry, Congruence & Similarity, Transformations I, and Transformations II. The last unit, dealing with solid and projective geometries, has the remaining four lessons: Prisms & Cylinders, Pyramids & Cones, Spheres, and Projective Geometry. The lessons appear in Appendix A.

The Lesson Plans

In preparing the lesson plans, Presmeg's prescription that "the mathematics curriculum should incorporate elements of the cultural histories of all the people of the region and should be experienced as 'real' by all children, and should resonate, as far as possible, with diverse home cultures" was taken into account (Presmeg, 1989, p. 172).

In writing the lesson plans, the format used was due primarily to Posamentier and Stepelman (Teaching Secondary School

<u>Mathematics</u>, 1990). An outline of the format is given below.

TOPIC:

PREVIOUSLY LEARNED KNOWLEDGE:

AIM:

MOTIVATION:

DO-NOW EXERCISE:

DEVELOPMENT AND METHODS:

DRILL:

MEDIAL SUMMARY:

APPLICATIONS AND DRILL:

FINAL SUMMARY AND CONCLUSION:

HOMEWORK ASSIGNMENT:

SPECIAL EQUIPMENT NEEDED:

IF TIME:

REFERENCES:

This lesson plan format was chosen because it systematically outlines steps many experienced teachers follow in order to teach lessons. Also, this format is known to many urban teachers, and, in particular, to teachers in the chosen school where the field trial materials occurred. Books and resources, including those sources mentioned in Chapter II, were consulted for the lesson plans, teaching techniques, and teaching aids.

Aside from teaching with a multicultural approach, the experimental lessons included other instructional variations. There were instances when lecture was used (an example is the lesson on Angles); there were times when cooperative learning and mixed-ability grouping were used (examples are the lessons on Right Triangles and Transformations). Some lessons called for a hands-on approach (an example is the lesson on Spheres), whereas others used manipulatives and/or visual aids (an example is the lesson on Polygons).

For the homework, the investigator also used a variety of forms. The homework ranged from solving pencil-and-paper exercises to drawing (an example is the lesson on Angles) or sketching a figure (an example is the lesson on Quadrilaterals); from creating designs and patterns (an example is the lesson on Symmetry) to building a model (an example is the lesson on Prisms & Cylinders). There were instances where students were asked give a narrative description of their work (an example is the lesson on

Triangles).

The Pre-Assessment and Post-Assessment

To determine the background mathematical knowledge of the students, the investigator developed a pre-assessment instrument for each unit. In the pre-assessment, three questions on each lesson of the unit were asked. The chosen format for the pre-assessment was multiple-choice, with four choices for each question. The pre-assessment of Unit I had 21 questions; Unit II had nine questions; Unit III had nine questions; and Unit IV had nine questions. Since Unit III has two lessons on the same topic, Transformations, there were only three questions for this topic. Also in Unit IV, no mathematics question pertaining to Projective Geometry was asked, resulting in only nine questions. The lesson on Projective Geometry was considered to be more of an extension of Solid Geometry and an application in the Arts, rather than a mathematical concept. The students were to complete these pre-assessments prior to the teaching of each unit.

The investigator also created a post-assessment instrument for each unit. The mathematical questions asked on the post-assessments were the same as in the pre-assessments and the testing format was also the same. Further, to determine if the students would remember the multicultural aspects of the lessons, a multicultural section was included in each of the post-assessments. Two questions for each lesson were asked. Hence, the first part of the post-assessment dealt with the mathematical aspect and the second part dealt with the multicultural aspect. To vary the format, the multicultural section was in the form of fill-in-the-blanks. Therefore, the post-assessment for Unit I had 21 questions in the mathematics section and 14 questions in the multicultural section; the post-assessment for Unit II had nine questions in the mathematics section and six questions in the multicultural section; the post-assessment for Unit III had nine questions in the mathematics section and six questions in the multicultural section; and the post-assessment for Unit IV had nine questions in the mathematics section and eight questions in the multicultural section. As noted before, even if there were four lessons in Unit III, only nine mathematics questions and six multicultural questions were asked since there were two lessons for Transformations.

Since the lesson on Projective Geometry was taught with a multicultural approach, there were two questions asked regarding this topic in the multicultural section, giving a total of eight multicultural questions of the post-assessment. The students were to complete these post-assessments after the teaching of each unit.

Both the pre-assessments and the post-assessments appear in Appendix B.

The Student Questionnaire

To acquire feedback about the curricular materials from the students, the investigator created a questionnaire for each unit. Each questionnaire was to be completed after the entire unit had been taught. The questionnaire format was the same for the four units, varying only with regard to the mathematical topics addressed. A total of nine questions appeared in each questionnaire and the questions are listed below.

Question 1: I acquired a good understanding of the topic listed below while learning about a direct application of the topic in another culture. For this question, the students were given a scale from 1 - 5, with 1 being the lowest and 5 being the highest. They were to circle their answer to this statement for the different topics in the given unit. Each of the mathematical topics for the unit was listed after the question.

Question 2: I enjoyed the topic listed below while learning about a direct application of the topic in another culture. As in Question 1, students were given a scale from 1 - 5, with 1 being the lowest and 5 being the highest, with answers sought for each mathematical topic in the unit. Again, the different mathematical topics covered in a unit were listed.

Question 3: For each of the topics below, write at least two direct applications or uses of the topic in your own or other people's surroundings. The students were asked to write their responses. This question enabled students to give multicultural applications other than those used in teaching the lessons. Again, each mathematical topic of the unit was listed under this question.

Question 4: Direct applications of the lessons provided me with a better understanding of other cultures. Once again, the students were given a scale from 1 - 5, with 1 being the lowest and

5 being the highest. Each student was to circle his/her choice.

Question 5: The lessons involving direct applications provided me with better appreciation of other cultures. Again, a scale from 1 - 5, with 1 being the lowest and 5 being the highest, was used to respond to this question.

Question 6: What topic or lesson did you like most? Why? The students were to write their responses to this question.

Question 7: What topic or lesson did you like least? Why? Again, for this question, the students were to write their responses.

Question 8: What was your favorite lesson? Why? For this question, the students were to write their responses.

Question 9: Write any comments or suggestions to improve the approach regarding this unit. Students were to write their responses to this question. This question sought for the students' opinions and criticisms for the unit.

The student questionnaire for each unit is found in Appendix C.

Evaluation Form

After consulting Vogeli's Evaluation Guide for Reviewers

(1992) that he used for jurors to evaluate his own lessons, the investigator developed an evaluation form consistent with the needs of this study. This form guided the members of the jury in evaluating the lesson plans. A total of nine questions were asked in the evaluation form. The questions are as follows.

Question 1 asked the evaluator to give his/her name for record purposes.

Question 2 asked the evaluator for the grade level(s) for which each lesson was suitable. The lessons, classified according to units, were all listed after the question.

Question 3 requested for the evaluator's opinion regarding the sequencing of the lessons.

Question 4 asked the evaluator to list ways to improve the lessons for better use by other teachers. Once again, the topics were all listed after the question.

Question 5 asked for ways to make the lessons more interesting to teachers. This question was asked to solicit different ideas about how teachers might use the lessons in their classrooms.

Question 6 asked for means to make the lessons more interesting to students. This question sought ideas about making the lessons more appealing to students.

Question 7 requested the evaluator's opinion about whether or not the lessons foster awareness, appreciation, and acknowledgment of other cultures. The topics list accompanied this question.

Question 8 asked the evaluator to rate the subject matter, the pedagogy, and the cultural awareness of each of the topic. In this particular question, a scale from 1 - 5, with 1 being the lowest and 5 being the highest, was used. The evaluator had to write the corresponding number for his/her response.

Question 9 asked for comments, suggestions, and recommendations for the lessons.

The Evaluation Form appears in Appendix D.

The School and the Students

The school chosen for the try-out of the curricular materials was an independent school in New York City. The school, which had a student-body representing at least 105 countries, seemed a receptive setting for use of multicultural approach in teaching geometry in the middle grades.

The investigator was assigned two classes in which geometry could be taught using the developed lesson plans and was given six weeks to conduct the study. A total of 46 students participated in the study. The students came from various ethnic, cultural, social, and economic backgrounds and more than half (at least 60%) came from a household whose first language was not English. The classes were mixed in ability and there appeared to be a balance between female and male students.

The academic year was in its fifth month and the students had already finished Numeration, Fractional Concepts, Percentage, and Ratio and Proportion. All the students had learned some geometric concepts like points, lines, etc. and some measurements like length of a line segment and degrees in an angle in their earlier grades.

The Administration of the Study

The investigator met with the students before the start of the study to introduce the teaching approach to be used and the topics to be learned. At the start of Unit I, the students were asked first to complete the pre-assessment for the entire unit. The pre-assessments were checked by the investigator and were returned to the students before any lesson was taught. Then the lessons of the unit were taught. A new lesson/topic was taught each day and at the end of each lesson, homework was given. The students submitted their homework for all the lessons after the entire unit had been taught. Submitting homework at the end of the unit gave the students more time to work on the homework and served as a review for the whole unit. After finishing a unit, the students were given an extra day for review and to ask questions. The next day, they were then given the post-assessment. The investigator checked the post-assessments and returned them to the students the following meeting. Next, the students also were asked to complete a questionnaire at the end of each unit. They completed the questionnaires before seeing the results of the post-assessment. The process was repeated for the remaining three units until the study ended; no break between units was given.

Unit I, being the longest, took ten school days -- from the pre-assessment to the post-assessment, including the submission of the homework and the completion of the student questionnaire. Unit II took six school days, Unit III took seven days, and finally Unit IV was completed during the remaining seven days.

During the course of the study, the investigator maintained a daily log of his own observations on the teaching of the lessons, the students' reactions to the lessons, the general classroom atmosphere, and the students who were absent.

The students then were interviewed in groups of five or six. The interviews enabled the students to act as a focus group with regard to a multicultural approach in teaching middle grades geometry. Due to school holidays and the time it took the investigator to analyze the responses to the questionnaires, a period of two months was needed before the interviews finally took place. The interviews were held on school grounds during normal school hours. The students were asked to speak their minds and no structured questioning occurred. Their opinions and interview responses were recorded in the log of the investigator.

The curricular materials were then given to five educators for their evaluation. Two of the professionals were middle school teachers of mathematics. Both these teachers were employed in the school where the study was conducted. The remaining three members of the jury were experts in the field of multicultural mathematics and ethnomathematics. Of this five-member jury, four are female and four are Americans. The jurors were given copies of the lesson plans for evaluation and were asked to complete the evaluation form to critique the lesson plans.

Chapter IV

RESULTS OF THE STUDY

Synopsis of the Study

After surveying the literature on multicultural education and some middle grades mathematics textbook series, the investigator wrote 18 geometry lessons using a multicultural approach. These lessons were designed to replace portions of a middle grades geometry curriculum dependent upon standard textbooks and were piloted in an independent New York City school. The study lasted for six weeks and 46 students participated in the study.

The lessons were divided into four units and, at the start of each unit, the students were given a pre-assessment on the mathematical topics. After the entire unit had been taught, the students completed a post-assessment on both the mathematical and the cultural topics. Additionally, they were asked to complete a

questionnaire and were interviewed. The investigator maintained a daily log of his observations throughout the field trial. Finally, a five-member jury reviewed the lessons and completed an evaluation form supplied by the investigator.

Chapter Format

This chapter contains the results of the study, based on the responses from the student questionnaire, from the interview, and from the jury evaluation. The first part of this chapter shows a unit by unit tabular report of the pre-assessment and post-assessment; a tabular report on the students' questionnaire responses; and a narrative report on the investigator's observations and interviews with students. The second part of the chapter is a report on the jurors' evaluation. The last part of the chapter is a discussion of the results.

Report on Students' Responses

Unit I Geometric Concepts

Unit I consists of seven lessons and deals with basic geometric concepts. The lessons in this unit are: Angles, Parallel & Perpendicular Lines, Transversals, Triangles, Quadrilaterals, Polygons, and Circles.

Student questionnaire responses follow next. The response to question 1 is displayed in Table 2.

On a scale of 1 - 5, the students were then asked whether they enjoyed the topic while learning about a direct application of the topic in another culture. The results are shown in Table 3.

Table 1

Mean Scores for the Pre-Assessment and Post-Assessment (n=46)

ASSESSMENT	MEAN SCORE	STANDARD DEVIATION
Pre-Assessment [a]		
Mathematics Section	8.85	3.51
Post-Assessment [b]		
Mathematics Section	14.33	3.27
Multicultural Section	5.54	2.44

Note. [a] The mathematics section of the pre-assessment contained 21 questions. [b] The mathematics section of the post-assessment contained 21 questions while the multicultural section contained 14 questions.

Table 2

Student Responses to: I acquired a good understanding of the topics while learning about a direct application of the topic in another culture. (n = 46)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
ANGLES	0	1	9	16	19	1
PARALLEL & PERPENDICULAR LINES	0	5	8	18	14	1
TRANSVERSALS	1	9	10	18	6	2
TRIANGLES	0	1	1	16	25	3
QUADRILATERALS	0	1	9	15	19	2
POLYGONS	0	2	8	18	16	2
CIRCLES	0	4	5	19	16	2

Note. On the response scale 1 is the lowest and 5 is the highest.

Table 3

Student Responses to: I enjoyed the topics while learning about a direct application of the topic in another culture. (n = 46)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
ANGLES	1	1	14	9	19	2
PARALLEL & PERPENDICULAR LINES	2	5	14	10	14	1
TRANSVERSALS	3	6	15	12	9	1
TRIANGLES	1	1	5	13	24	2
QUADRILATERALS	1	5	6	11	21	2
POLYGONS	0	4	8	14	17	3
CIRCLES	3	6	4	15	17	1

Note. On the response scale 1 is the lowest and 5 is the highest.

When the 46 students were asked to give direct applications or uses of the topic in their own or other people's surroundings, 32 were able to do so for Angles (Examples were windows, buildings, etc.); 35 for Parallel & Perpendicular Lines (Examples were streets, gymnastics, skiing, railroad tracks, etc.); 26 for Transversals (Examples were Tic-Tac-Toe, baskets, etc.); 32 for Triangles (Examples were pizzas, cake slices, etc.); 31 for Quadrilaterals (Examples were checkers and chess boards, windows, TV, etc.); 26 for Polygons (Examples were bathtubs, buildings, etc.); and 33 for Circles (Examples were buttons, wheels, pies, clocks, etc.).

The students were also asked if (a) direct applications of the lessons provided them with a better understanding of other cultures and (b) the lessons involving direct application provided them with a better appreciation of other cultures. The responses are listed in Table 4.

The questionnaire also asked which topic students liked most; liked least; and which was their favorite. The lesson that was liked most was Parallel & Perpendicular Lines (11 out of 46); Transversals was liked the least (12 out of 46); and Parallel & Perpendicular Lines was the favorite of the students (11 out of 46).

Finally, 26 students were able to write comments on the unit. Some positive comments were: "This is a direct and good approach," "Give more time to the lessons and cultural examples," "It was fun and useful," and "I liked the unit." Some negative comments were: "No drawing," and "Use things known to us."

Log entries by the investigator indicated that the students appeared to be excited at the start of this unit. They were usually attentive and cooperative in doing the work. Of all the lessons in this unit, Parallel & Perpendicular Lines was the one that held the classes captive until the end of the lesson. The students were most cooperative and most attentive during this particular lesson. The classes struggled most in understanding the lesson on Transversals. A majority of the students had difficulties in doing the assigned homework, giving a reason that the directions of the homework were unclear. Also towards the middle of the unit, some students remarked that they were not doing mathematics, since they did not deal too much with numbers and they had to write words and stories. Regardless of the complaints, the students completed their homework.

Table 4

Responses of Students to Questions: Direct application of the lessons provided me with (a) a better understanding of other cultures, and (b) a better appreciation of other cultures. (n = 46)

QUESTION	RESPONSES 1	2	3	4	5	NO RESPONSE
a)Better understanding of other cultures	0	1	14	17	14	0
b)Better appreciation of other cultures	1	2	10	14	18	1

Note. On the response scale 1 is the lowest and 5 is the highest.

When the students were answering the questionnaire, many students remarked that Question 6 (most liked lesson) and Question 8 (favorite lesson) were the same. The investigator explained that the most liked lesson was the one that pleased them most in the unit using a multicultural approach while the favorite lesson was the one that they enjoyed doing most, regardless of the teaching approach utilized.

<u>UNIT II</u> Geometric Measurements

Unit II has a total of three lessons -- Right Triangles, Perimeter & Area of Polygons, and Circumference & Area of Circles.

In Table 5, the mean scores of the pre-assessment and the post-assessment are displayed. The figures indicated an increase of about 37% from the pre-assessment to the post-assessment.

Student questionnaire responses follow next. The response to question 1 is displayed in Table 6.

On a scale of 1 - 5, the students were then asked whether they enjoyed the topic while learning about a direct application of the topic in another culture. The results are shown in Table 7.

Table 5

Mean Scores for the Pre-Assessment and Post-Assessment (n=46)

ASSESSMENT	MEAN SCORE	STANDARD DEVIATION
Pre-Assessment [a]		
Mathematics Section	3.50	1.41
Post-Assessment [b]		
Mathematics Section	4.78	1.84
Multicultural Section	2.87	1.64

Note. [a] The mathematics section of the pre-assessment contained nine questions. [b] The mathematics section of the post-assessment contained nine questions while the multicultural section contained six questions.

Table 6

Student Responses to: I acquired a good understanding of the topics while learning about a direct application of the topic in another culture. (n = 46)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
RIGHT TRIANGLES	1	1	10	16	16	2
PERIMETER & AREA OF POLYGONS	1	2	6	24	11	2
CIRCUMFERENCE & AREA OF CIRCLES	0	5	8	20	10	3

Note. On the response scale 1 is the lowest and 5 is the highest.

Table 7

Student Responses to: I enjoyed the topics while learning about a direct application of the topic in another culture. (n = 46)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
RIGHT TRIANGLES	4	3	9	16	12	2
PERIMETER & AREA OF POLYGONS	4	2	11	18	9	2
CIRCUMFERENCE & AREA OF CIRCLES	4	2	9	15	13	3

Note. On the response scale 1 is the lowest and 5 is the highest.

When asked to give direct applications or uses of the topic in their own or other people's surroundings, 34 out of 46 students were able to do so for Right Triangles (Examples were sails, paper, airplanes, etc.); 34 for Perimeter & Area of Polygons (Examples were paper, buildings, landscapings, etc.); and 29 for Circumference & Area of Circles (Examples were Ferris Wheel, tires, clocks, etc.).

The students were also asked if (a) direct applications of the lessons provided them with a better understanding of other cultures and (b) the lessons involving direct application provided them with a better appreciation of other culture. The responses are listed in Table 8.

The questionnaire also asked which topic students liked most; liked least; and which was their favorite. The lesson that was liked most was Perimeter & Area of Polygons (15 out of 46); Right Triangles was liked the least (11 out of 46); and Perimeter & Area of Polygons was the favorite of the students (15 out of 46). Finally, 25 out of the 46 students wrote comments on the unit. Most of the comments (36 out of 46) asked for more examples and more cultures and indicated that the lessons were fun and good. One student commented that art should not be incorporated in mathematics.

Table 8

Responses of Students to Questions: Direct application of the lessons provided me with (a) a better understanding of other cultures, and (b) a better appreciation of other cultures. (n = 46)

QUESTION	RESPONSES 1	2	3	4	5	NO RESPONSE
a)Better understanding of other cultures	1	3	11	25	6	0
b)Better appreciation of other cultures	2	4	12	14	14	0

Note. On the response scale 1 is the lowest and 5 is the highest.

In this unit, log entries indicated that students were not as enthusiastic as before, perhaps due to the length of the unit. They showed excitement in reenacting the Egyptian Rope Stretchers but showed difficulty in solving for the third number in a Pythagorean Triple. Both the homework on drawing an animal using triangles and quadrilaterals only and on creating their own Mandala interested the students greatly. "I enjoyed making my own designs!" was a common declaration about the unit.

UNIT III Geometric Transformations

Unit III, dealing with geometric transformations, has four lessons: Symmetry, Congruence & Similarity, Transformations I, and Transformations II.

In Table 9, the mean scores of the pre-assessment and the post-assessment are displayed. The figures indicated an increase of about 76% from the pre-assessment to the post-assessment.

Student questionnaire responses follow next. The response to question 1 is displayed in Table 10.

Table 9

Mean Scores for the Pre-Assessment and Post-Assessment (n=46)

ASSESSMENT	MEAN SCORE	STANDARD DEVIATION
Pre-Assessment [a]		
Mathematics Section	4.11	1.62
Post-Assessment [b]		
Mathematics Section	7.24	1.30
Multicultural Section	3.65	1.32

Note. [a] The mathematics section of the pre-assessment contained nine questions. [b] The mathematics section of the post-assessment contained nine questions while the multicultural section contained six questions.

Table 10

Student Responses to: I acquired a good understanding of the topics while learning about a direct application of the topic in another culture. (n = 46)

	RESPONSES					
TOPIC	1	2	3	4	5	NO RESPONSE
SYMMETRY	1	0	9	17	17	2
CONGRUENCE & SIMILARITY	1	2	8	18	15	2
TRANSFORMATIONS	1	1	12	21	9	2

Note. On the response scale 1 is the lowest and 5 is the highest.

On a scale of 1 - 5, the students were then asked whether they enjoyed the topic while learning about a direct application of the topic in another culture. Table 11 shows the results.

When asked to give direct applications or uses of the topic in their own or other people's surroundings, 34 students were able to do so for Symmetry (Examples were hub caps, pies, butterflies, etc.); 31 for Congruence & Similarity (Examples were map making, architecture, picture enlargements, etc.); and 29 for Transformations (Examples were Tartans, quilt patterns, etc.).

The students were also asked if (a) direct applications of the lessons provided them with a better understanding of other cultures and (b) the lessons involving direct application provided them with a better appreciation of other culture. The responses are in Table 12.

The questionnaire also asked which topic students liked most; liked least; and which was their favorite. The lesson that was liked most was Transformations (17 out of 46); Symmetry was liked the least (13 out of 46); and Transformations was the favorite of the students (20 out of 46). Finally, 26 of the 46 students wrote comments on the unit. Those who wrote comments indicated that there should be more varieties of culture and more applications.

Table 11

Student Responses to: I enjoyed the topics while learning about a direct application of the topic in another culture. (n = 46)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
SYMMETRY	2	2	12	16	11	3
CONGRUENCE & SIMILARITY	1	3	8	20	12	2
TRANSFORMATIONS	1	4	7	21	11	2

Note. On the response scale 1 is the lowest and 5 is the highest.

Table 12

Responses of Students to Questions: Direct application of the lessons provided me with (a) a better understanding of other cultures, and (b) a better appreciation of other cultures. (n = 46)

QUESTION	RESPONSES 1	2	3	4	5	NO RESPONSE
a)Better understanding of other cultures	1	1	13	22	7	2
b)Better appreciation of other cultures	1	5	8	18	12	2

Note. On the response scale 1 is the lowest and 5 is the highest.

In the third unit, the excitement of the students appeared to return. They demonstrated cooperation in all the lessons. Although some students experienced difficulty in understanding "rotational symmetry," other classmates helped them out. In the Transformations (II) lesson, the students enjoyed finding a basic pattern for a particular design and they also enjoyed the homework. It was a favorable unit.

UNIT IV Solid and Projective Geometries

This unit consists of four lessons: Prisms & Cylinders, Pyramids & Cones, Spheres, and Projective Geometry.

In Table 13, the mean scores of the pre-assessment and the post-assessment are displayed. The figures indicated an increase of about 90% from the pre-assessment to the post-assessment.

Student questionnaire responses follow next. The response to question 1 is displayed in Table 14.

On a scale of 1 - 5, the students were then asked whether they enjoyed the topic while learning about a direct application of the topic in another culture. The results are displayed in Table 15.

When asked to give direct applications or uses of the topic in their own or other people's surroundings, 42 students were able to do so for Prisms & Cylinders (Examples were cans, buildings, etc.); 40 for Pyramids & Cones (Examples were ice cream cones, dunce cap, etc.); 41 for Spheres (Examples were balls, globes, Matzoh balls, etc.); and 34 for Projective Geometry (Examples were paintings, railroad tracks, etc.).

The students were also asked if (a) direct applications of the lessons provided them with a better understanding of other cultures and (b) the lessons involving direct application provided them with a better appreciation of other culture. The responses are listed in Table 16.

The questionnaire also asked which topic students liked most; liked least; and which was their favorite. The lesson that was liked most was Prisms & Cylinders (17 out of 46); Projective Geometry was liked the least (15 out of 46); and Prisms & Cylinders was the favorite of the students (14 out of 46). Finally, 24 out of the 46 students wrote comments on the unit. The students' comments varied from the positive to the negative. A majority of them (35 out

Table 13

Mean Scores for the Pre-Assessment and Post-Assessment (n=46)

ASSESSMENT	MEAN SCORE	STANDARD DEVIATION
Pre-Assessment [a]		
Mathematics Section	2.46	1.15
Post-Assessment [b]		
Mathematics Section	4.67	1.79
Multicultural Section	4.00	2.13

Note. [a] The mathematics section of the pre-assessment contained nine questions. [b] The mathematics section of the post-assessment contained nine questions while the multicultural section contained eight questions.

Table 14

Student Responses to: I acquired a good understanding of the topics while learning about a direct application of the topic in another culture. (n = 46)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
PRISMS & CYLINDERS	1	4	9	20	11	1
PYRAMIDS & CONES	1	7	11	18	8	1
SPHERES	2	4	15	18	6	1
PROJECTIVE GEOMETRY	0	7	12	15	9	3

Note. On the response scale 1 is the lowest and 5 is the highest.

Table 15

Student Responses to: I enjoyed the topics while learning about a direct application of the topic in another culture. (n = 46)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
PRISMS & CYLINDERS	3	6	11	16	9	1
PYRAMIDS & CONES	3	5	12	14	11	1
SPHERES	6	5	16	9	9	1
PROJECTIVE GEOMETRY	4	5	10	19	6	2

Note. On the response scale 1 is the lowest and 5 is the highest.

Table 16

Responses of Students to Questions: Direct application of the lessons provided me with (a) a better understanding of other cultures, and (b) a better appreciation of other cultures. (n = 46)

QUESTION	RESPONSES 1	2	3	4	5	NO RESPONSE
a)Better understanding of other cultures	1	6	12	18	8	1
b)Better appreciation of other cultures	2	4	14	11	14	1

Note. On the response scale 1 is the lowest and 5 is the highest.

of 46) remarked that the lessons were good; others (8 out of 46) said they were boring. Again, more than half (30 out of 46) asked for more cultural examples. One suggested that the class should go on a field trip.

This unit, according to the investigator's log entries, produced minimal enthusiasm and cooperation from the classes. They were not as enthusiastic as before. Perhaps it could be attributed to the complexity of the subject matter and the different formulas students have to remember. Students enjoyed the visual aids depicting different kinds of solid geometric shapes. These manipulatives appeared to have transformed an abstract concept into a concrete one. The lesson on Projective Geometry was taught unsuccessfully, resulting in the cancellation of the homework. The investigator intended to draw a figure that would produce a 3-D effect. After several attempts to draw the figure, the investigator was not able to produce the 3-D figure. The classes were extremely uncooperative during this particular lesson. However, it was here in Unit IV that the students were able to give the most responses to the application of the topics in their own or other people's surroundings. When asked in the interviews why this was so, most students (39 out ot 46) simply answered that they see these figures in their own lives and they are familiar with them. Additionally, much of the students' focus was upon the memorization of the formulas for volume, lateral areas and surface areas of solids.

When the students were interviewed, they gave a wide range of comments. More than two-thirds of the students (35 out of 46) said that they enjoyed learning about other cultures; they could see the applications and uses of mathematics; the lessons were more interesting, more fun and more real; and there was some sense of pride involved. Negative comments included taking instructional time from mathematics; lack of review time; and having a topic a day is unconventional. One noted suggestion was to make the multicultural segment as an enrichment instead of a requirement.

Report on Jury Evaluation

A five-member jury was asked to evaluate the curricular materials. The jury was comprised of four females and one male, two of whom were employed in the school where the materials were tested. The other three members of the jury were experts in multicultural mathematics. Also, one of the jury members was Brazilian. (This juror was chosen for his/her expertise in multicultural mathematics and not for his/her nationality.) The evaluation results follow.

One juror felt that the lessons in this study were appropriate for grade 7, but some materials also were suitable for 6th and 8th grades. Another juror also indicated that the materials were suitable for 7th grade, and some lessons were appropriate for the 6th and 8th grades. A third member of the jury gave a span from 5th grade to the 12th grade for all the lessons. A fourth juror felt that the lessons in the first unit were for 2nd - 4th grades, the lessons in the second unit were for 5th - 6th grades, the lessons in the third unit were for 6th grades, and the lessons for the fourth unit were for 6th - 7th grades. The last member of the jury did not specify any grade levels for the curricular materials claiming that appropriate grade levels were dependent upon the previous experience of students and that choosing a particular grade level was not appropriate.

With regard to the sequencing of the lessons, four agreed that there was a nice flow of topics and that the sequencing was adequate. A fifth juror stated that the sequencing of the topics moved from less difficult to more difficult.

To enable better use by other teachers, all jurors felt that there should be more cultural and historical background for each lesson. One mentioned that the background material must be readily available for teachers to use, since otherwise teachers might not have the necessary time or inclination to access such resources. This juror also suggested that the lessons be integrated with other disciplines. One juror requested more drill problems in the lesson plans. The responses of the other jury members on how to make the lessons more interesting to teachers were consistent with the suggestions above.

Suggestions varied in response to the question on how to make the lessons more interesting to students. One juror suggested that the students be informed completely of the cultural aspects of the lessons, in the same manner that the teachers were to be supposedly informed. Another suggested continuation of hands-on homework and projects. Finally, one suggested maintaining the active involvement of the students in the lessons since, according to the juror, this is the manner in which people learn best.

All the jurors agreed that the curricular materials as written foster awareness, appreciation and acknowledgment of other cultures. According to one juror, "excellent cultural examples were used to illustrate the mathematical topics." Another juror noted that some intricate patterns led to "an appreciation of their level of development." One jury member asserted that some lessons need to have more detailed cultural information in order to foster awareness, appreciation and acknowledgment of other cultures.

The average of the ratings on (I) the subject matter, (II) the pedagogy, and (III) the cultural awareness of each topic are given in Tables 17, 18 and 19.

Table 17

Ratings of Jurors on the Subject Matter of the Topics (n=5)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
ANGLES	0	0	2	0	2	1
PARALLEL & PERPENDICULAR LINES	0	0	1	1	2	1
TRANSVERSALS	0	0	2	0	2	1
TRIANGLES	0	0	1	1	2	1
QUADRILATERALS	0	0	1	1	2	1
POLYGONS	0	0	2	1	1	1
CIRCLES	0	0	0	2	2	1
RIGHT TRIANGLES	0	0	0	2	2	1
PERIMETER & AREA OF POLYGONS	0	0	1	1	2	1
CIRCUMFERENCE & AREA OF CIRCLES	0	0	1	2	1	1
SYMMETRY	0	0	0	3	1	1
CONGRUENCE & SIMILARITY	0	0	0	3	1	1
TRANSFORMATIONS	0	0	0	2	2	1
PRISMS & CYLINDERS	0	0	0	1	3	1
PYRAMIDS & CONES	0	0	0	1	3	1
SPHERES	0	0	0	0	4	1
PROJECTIVE GEOMETRY	0	0	0	0	4	1

Note. On the response scale 1 is the lowest and 5 is the highest.

Table 18

Ratings of Jurors on the Pedagogy of the Topics (n=5)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
ANGLES	0	0	2	1	1	1
PARALLEL & PERPENDICULAR LINES	0	0	1	1	2	1
TRANSVERSALS	0	0	2	0	2	1
TRIANGLES	0	0	1	2	1	1
QUADRILATERALS	0	0	1	1	2	1
POLYGONS	0	0	1	2	1	1
CIRCLES	0	0	0	3	1	1
RIGHT TRIANGLES	0	0	1	2	1	1
PERIMETER & AREA OF POLYGONS	0	0	1	1	2	1
CIRCUMFERENCE & AREA OF CIRCLES	0	0	1	2	1	1
SYMMETRY	0	0	0	2	2	1
CONGRUENCE & SIMILARITY	0	0	0	3	1	1
TRANSFORMATIONS	0	0	1	1	2	1
PRISMS & CYLINDERS	0	0	0	2	2	1
PYRAMIDS & CONES	0	0	0	0	4	1
SPHERES	0	0	1	0	3	1
PROJECTIVE GEOMETRY	0	0	0	0	4	1

Note. On the response scale 1 is the lowest and 5 is the highest.

Table 19

Ratings of Jurors on the Cultural Awareness of the Topics (n=5)

TOPIC	RESPONSES 1	2	3	4	5	NO RESPONSE
ANGLES	0	1	1	1	1	1
PARALLEL & PERPENDICULAR LINES	0	0	1	1	2	1
TRANSVERSALS	0	1	1	0	2	1
TRIANGLES	0	0	1	2	1	1
QUADRILATERALS	0	0	1	1	2	1
POLYGONS	0	0	2	1	1	1
CIRCLES	0	0	1	1	2	1
RIGHT TRIANGLES	0	0	1	2	1	1
PERIMETER & AREA OF POLYGONS	1	0	0	1	2	1
CIRCUMFERENCE & AREA OF CIRCLES	0	0	1	1	2	1
SYMMETRY	0	0	0	1	3	1
CONGRUENCE & SIMILARITY	0	0	0	3	1	1
TRANSFORMATIONS	0	0	0	3	1	1
PRISMS & CYLINDERS	0	0	1	0	3	1
PYRAMIDS & CONES	0	0	0	1	3	1
SPHERES	0	0	1	0	3	1
PROJECTIVE GEOMETRY	0	0	0	0	4	1

Note. On the response scale 1 is the lowest and 5 is the highest.

Some specific juror recommendations are indicated below. According to one juror, instead of giving multiple-choice or fill-in-the-blanks test formats, questions that "require thinking about why various peoples solved their mathematical problems in certain ways" was a much preferred test format. This juror approved of the student questionnaire, particularly the questions about cultural applications, and stated that sources for any quotation or illustration must be indicated. Another juror commented that more background materials (e.g. National Geographic), not necessarily focusing on mathematics but on culture, history, geography, etc., should be provided. According to this juror, such materials enhance cultural understanding and lead to mathematics helping cultural understanding, and, in a sense, become a form of applied mathematics. One last recommendation of this juror was to interview the teachers and the parents to determine the impact at home of this way of doing mathematics. Another juror offered references to be included in the references section and suggested that the students should develop the formulas and that the use of hands-on activities and visual aids should be maintained. Lastly, some jurors requested that certain terms be changed or removed, since others might find the terms derogatory and insulting, and that some instructions and sentences be reworded for clarity.

In conclusion, all the jurors reported satisfaction with the curricular materials. They agreed that the curricular materials fostered cultural awareness and three out of the five jurors agreed that most of these lessons were appropriate for the middle grades.

Discussion

One noticeable result of this study was the unimpressive student performances in the mathematics sections of the post-assessments. Of the four units, it was in Unit III (Geometric Transformations) that the class averaged 80% (7.24 out of 9); the remaining three units showed mediocre performances-- Unit I (Geometric Concepts), average 68% (14.33 out of 21); Unit II (Geometric Measurements), average 53% (4.78 out of 9); and Unit IV (Solid and Projective Geometries), average 52% (4.67 out of 9). The investigator sought results of standardized tests in middle grades geometry for comparison with the study's results; however, no materials were available. A possible explanation for this circumstance could be attributed to the students' lack of reinforcement and retention of the topics. As noted before, a brand new topic was introduced each day and only a day was given for the general review of the entire unit. It has been the experience of the investigator that students needed constant strengthening and support of the lessons. This could also explain why the students scored very poorly in the cultural sections-- Unit I, average 40% (5.54 out of 14); Unit II, average 48% (2.87 out of 6); Unit III, average 61% (3.65 out of 6); and Unit IV, average 50% (4 out of 8). There should had been constant reinforcement and extension of the cultural topics, perhaps in other disciplines since the schedule limited the teaching time of these topics in mathematics. From the pre-assessment to the post-assessment, the scores increased in all four units. Two reasons could serve as explanations-- the lessons served as reviews from previously learned material and the students noted that the questions were the same for both the pre-assessment and post-assessment. The students remarked that the questions were similar in the first unit and did not mention it anymore in the subsequent units. It also was noted that Unit III produced the highest scores in both the mathematics and cultural sections compared to the other three units.

In doing the tabulation of the questionnaire, two observations were made by the investigator. When asked to give applications of the mathematical topics, all those who did gave applications in their own or other people's surroundings. They gave examples that were familiar and not unique to them and were parts of their own cultures. After all, the examples they had to give need not come from a foreign or exotic culture. This was a good question because the students saw real-world applications of mathematics outside the classroom and these applications were found in their own surroundings.

Another observation was that the students gave the same answers to two different questions-- one question asked for the favorite lesson while the other asked for the lesson most liked. It was brought to the attention of the insvestigator that these two questions were the same. The investigator pointed out that the favorite lesson was the lesson that pleased them most using a multicultural approach while the most liked lesson was the lesson they enjoyed most regardless of the teaching approach used. The students answered them without further questioning. Perhaps the students just assumed that these two were the same, which resulted to only one topic. Hence, it was essential that questions be worded very carefully in order that different interpretations of the questions would not occur. Also, questions had to be significant. Given the situation above, if it was the first time that a student was introduced to a brand new lesson, how would he/she know what his/her favorite lesson was regardless of the teaching approach used? Words and terminologies used in questionnaires must not confused the students. Making sure that they understood each question and words was important.

Of all the lessons in the study, log entries of the investigator indicated that the students were most attentive during the lesson for Parallel & Perpendicular Lines (Tartans). The students were most uncooperative during the lesson on Projective Geometry (Renaissance). An explanation for this occurrence was the fact that everyone was involved in the lesson. The lesson started with asking for a song that was very familiar to many of the students-- Old MacDonald. Here, the meaning of the last name was explained and the investigator volunteered the meaning of his last name. Students at this age group were very interested in knowing personal details about other people, particularly their teacher. The investigator asked the students to give their last names and define them. When everyone got involved, introducing the lesson to them was done smoothly. Unfortunately, this did not happen during the Projective Geometry lesson. The investigator, despite several attempts, was unsuccessful in drawing the figure that would produce the 3-D effect. This happened in both classes. Once the students saw that the lesson was going nowhere, their attentiveness immediately disappeared. During the other lessons, they were doing what was required of them but not with the same degree of enthusiasm and cooperation compared to the Perpendicular and Parallel Lines lesson. Working with groups and with manipulatives were enjoyed by the students. These lessons kept them involved and working throughout the entire class period. These lessons also gave them a chance to move around the classroom, express their thoughts and opinions, and help each other.

The comments students provided showed a great interest in using this approach and recommended it to other students. Aside from this approach, another factor needed to be considered. The students got a new teacher in the middle of the school year. Perhaps the students needed a break from their usual teachers and a fresh start was what they needed. Also, a multicultural approach dictated that the lessons be handled differently. Incorporating a cultural element in each of the lessons presented the lessons in a more practical manner. Interview results indicated that lectures were the least appreciated by the students. However, mathematics lessons with a cultural element delivered by a lecture method was more welcomed compared to lessons delivered by straight lecture. Students at this age group were, by nature, restless and could easily be distracted. They always should be presented with ideas to think about or be allowed to express their thoughts both in words and in writing. Multicultural approach was utilized because different cultures aroused their curiosities and enabled them to be more inquisitive. The more they asked, the more involved the whole class became. Also, the students saw a more concrete way of learning mathematics.

Chapter V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary of the Study

This investigation regarding the use of a multicultural approach in the teaching of middle grades geometry was based upon a review of the literature on multicultural education, development of 18 curricular lessons that were tried out in an independent school, responses of students to questionnaires and interviews, and evaluation by a five-member jury.

After surveying the literature on multicultural education and some middle grades mathematics textbook series, the investigator wrote 18 curricular lessons using a multicultural approach. These lessons were designed to replace portions of a middle grades geometry curriculum dependent upon standard textbooks. The investigator then used the lessons in an independent New York City school for field trial. The field trial lasted for a period of six weeks and a total of 46 students participated.

At the start of each unit, the students were given a pre-assessment on the mathematical topics. After the entire unit had been taught, the students completed a post-assessment on both the mathematical and the cultural topics. Additionally, they were asked to complete a questionnaire and were interviewed by the investigator to act a focus group regarding the use of a multicultural approach in the teaching of middle grades geometry. The investigator maintained a daily log of his observations throughout the field trial.

Finally, a five-member jury reviewed the 18 curricular lessons and completed an evaluation form supplied by the investigator.

Conclusions

This study on implementing a multicultural approach in the teaching of mathematics appeared to have supported Zaslavsky's (1990) claims that (1) students appreciate the contributions of cultures that are different from their own and (2) linking the study of mathematics with other disciplines and cultures provides more meaning to the mathematics studied. Evidence was provided by student questionnaires and in interviews. When asked why they enjoyed a multicultural approach, most of the students (29 out of 46) answered that they saw uses and applications of mathematics outside the classroom and in other cultures that they had not encountered in their previous mathematics classes. Also, the students realized that certain mathematics topics could be connected to other disciplines. Interview comments from the interview like "It was fun learning about mathematics and another culture at the same time," and "Math with other subjects is better," supported these claims.

The results of this study seemed consistent with the findings of Vogeli that a multicultural approach could " 'personalize' and 'naturalize' the otherwise abstract mental exercises of mathematical topics that are seldom related to everyday activities and considerations of the student" (Vogeli, 1992, p. 101). Again, many students indicated that they appreciated the mathematics topics more because they saw a direct and human way of applying the topics.

Finally, the results of this study appeared to support Nobre's

(1989) findings regarding students being highly motivated and more involved with the lessons when the mathematics in those lessons was related to the students' everyday lives. The investigator's log entries indicated that discussions and teaching of the mathematics lessons were lively because everyone was participating and providing input.

The study was conducted in order to answer some focus questions. The first question asked what cultures are appropriate for inclusion in the middle grades geometry curriculum. The responses of the students revealed that the culture used need not be an unfamiliar one as long as there is an application of the mathematical topic in that particular culture. In this study, no particular culture was identified; hence, the first focus question was not answered and is a limitation of this study.

The second focus question asked which lessons appear to help students learn the required mathematical skills. In this study, the students reported that hands-on activities and those activities that require them to create their own designs seemed to have helped them learn the topics. Examples given are creating the Mandala design and making a model of the sphere. Even though many of the students reported enjoying this approach and learning more about the topics, their post-assessments scores in both the mathematics and cultural sections denoted fair and unimpressive results. The investigator sought literature to compare these results with results of standardized tests in middle grades geometry, but no materials were available. Hence, this is another limitation of the study.

The third focus question asked for the lessons that appeal least (most) to the students. According to the students, lecture lessons were the least appealing to them, whereas those lessons requiring group work and visual aids were the most appealing. The log entries by the investigator noted that lectures tended to result in inactive and docile students whereas group work enlivened the classes. According to the students, all of the lessons were appealing because there were applications of the topics in the real-world. Somehow, what they conceived abstract before was now concrete. The cultural element in each lesson seemed to have established this connection.

The fourth focus question asked for the students' reactions to a multicultural approach in teaching middle grades mathematics. The students reacted positively, enjoyed this teaching approach and even recommended this approach to other students. In the questionnaire and the interview, they praised the clear connections between mathematics and other disciplines and the illustrations of mathematics in real-world situations.

The fifth focus question asked for the advantages or disadvantages of using a multicultural approach when teaching geometry. Students remarked that this approach incorporated other disciplines in their mathematics class and they now had a better appreciation of the mathematics topics and other cultures. Another advantage the investigator recognized was that the students were involved in doing their work. They worked throughout the classes and participation in the discussions were active. However, one noted disadvantage was the length of the class period. With many things to discuss, a typical 40-minute class period may not be enough for such lessons. Another possible disadvantage resided in the unfamiliar nature of the the multicultural content and the resulting possibility that the instructor might offer incorrect answers to student questions. Nelson (1993) identified this situation as one of the basic concerns accompanying a multicultural approach in teaching. This concern actually happened when the investigator implied that "Mac" was Scottish and "Mc" was Irish, when in fact both "Mac" and "Mc" were Gaelic in origin and "Mc" was simply a shortened version of "Mac." (See the lesson plan on Parallel and Perpendicular Lines in pp. 120 - 121 of Appendix A.)

The sixth focus question asked whether or not the students showed any appreciation for the different cultures after being taught using a multicultural approach. According to the students, they now saw different applications of mathematics, particularly geometry, in other cultures, and the students, through the questionnaires and interviews expressed an understanding, respect, and appreciation of other cultures.

Recommendations for the Improvement of the Materials

For the improvement of the materials, the jury recommends including more references with the lesson plans (including sources for the illustrations and sources for more background information regarding the specific cultures) and deleting some terminology that other cultures may find offensive, e.g. "tribe," "sacrificial altar."

The investigator recommends that directions/instructions and questions (for questionnaires, lesson plans, etc.) be worded carefully for easier and better understanding by users.

Recommendations for Researchers Doing a Similar Study

For researchers doing a similar study, the jury recommends eliminating the pre-assessment and changing the format of the post-assessment; to continue asking the students to complete the questionnaires, especially the questions about cultural applications in their own and other peoples' surroundings; and to extend the interview to include the teachers and even the parents of the students.

The investigator recommends videotaping each lesson. Reviewing each lesson before the next one enables a researcher to improve teaching methods and correct mistakes. Such videotaping may replace a daily written log.

Another recommendation is that the study be piloted first before the actual field study. In this manner, inconsistencies in the study may be identified and the study can be polished further before the field work itself begins.

Recommendations for Implementation by Educators

Based on this study's findings, recommendations for the implementation of these curricular lessons by educators are many. The jury recommends encouraging teachers to seek colleagues, parents, or even other professionals whose presentations and first-hand experiences may enrich the students' learning. The presentors' cultures, whether familiar or unfamiliar to the students, will give students many different perspectives in learning.

In implementing these lessons, the prospective user is urged to do further reading and research on the cultural topics concerned, since the lessons plans were meant to be utilized only as guides.

Another recommendation is to allot enough time for the planning and implementing of these lessons. As mentioned earlier, many students noted that a 40-minute period may not be enough for a particular topic. A prospective user needs to look carefully into his/her schedule to ensure that there is ample time to use the materials effectively, since overlapping into the next period may require an agreement with other teachers.

Recommendations for Further Research

Finally, recommendations for further research are as follows. An extension of this study would use a multicultural approach in the teaching of mathematics in curricular areas other than geometry. Some fields that may be used are Number Systems and Probability and Statistics. The many different cultures using mathematics other than geometry are abundant and need further research.

Another recommendation for further study is to investigate the possibilities of using a multicultural approach in the teaching of an interdisciplinary curriculum. In such cases, the opportunities to investigate the connections among the disciplines are greater.

This study focused on the middle grades. Another extension of this study is to look into the possibility of introducing a multicultural approach to the lower grades or upper grades. One jury member noted that some of the topics in this study are appropriate for the lower grades while another juror said that some topics would be used for upper grades.

The lessons in this study were tried by two classes in an independent school with a student body drawn from many cultures. The results from these two clases were incorporated into one report. Another potential investigation involves the comparison of the performances of any two classes, where one class uses a multicultural approach and one uses a more traditional method. This type of study might offer a different kind of insight into the advantages or disadvantages of using a multicultural approach in the teaching of mathematics. In the present study, the post-assessments scores of the students produced only fair results. It would be useful to compare the scores of a more traditional approach with those of a multicultural approach. Other suggestions include teaching more than two classes; conducting the study in a public school setting or in another school whose commitment to diversity and multiculturalism may not be as strong as in the school where the materials were tested (The school where the study was conducted has students representing at least 105 countries); involving teachers other than the investigator in teaching the lessons; and including parents and teachers as part of the study since their reactions to the lessons may give valuable input to the study.

One more recommendation for further investigation is the development of alternative assessments to evaluate the performance of the students. This investigator invoked the multiple-choice format for the pre-assessment and post-assessment in the mathematics section and the fill-in-the-blank format for the cultural section. One juror expressed some concern about these testing formats as they do not measure the students' performance accurately. Other investigators could develop another form of assessment while using a multicultural approach in the teaching of mathematics.

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Appendix A

LESSON PLANS

TOPIC: ANGLES

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines

AIM: 1) to name the different kinds of angles, 2) to define SUPPLEMENTARY and COMPLEMENTARY angles, 3) to use the sundial to identify angles

MOTIVATION: Ask the class what time the dismissal is. Have them illustrate it. Ask a volunteer what figure the hands form (ANGLE). Remind the class that the point where the hands meet is called the VERTEX and the hands can be considered as the SIDES of the angle. Next, ask the class what other instruments can be used to tell the time of day. Lead them into saying the "SUNDIAL."

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Say that both the Babylonians and the ancient Egyptians were using sundials to tell the time. Brainstorm with the class as to how a sundial works. Make sure that they realize that the sundial involves the sun, the stake and the shadows it casts. Show the transparency of the sundial. Mention that the sundial has to be positioned such that the stake is facing North. Ask where the sun rises (EAST). Re-enact the rising of the sun and then stop when it reaches 10:00 AM. Mark it and ask what figure is formed (ANGLE). Next, have the class predict what will happen when it is noon. Finally, mark the sundial upon reaching 4:00 PM. Using the East as one of the legs and the shadow of the stake as the other, have the students draw the three angles and have them describe them. Ask a volunteer what letter closely resembles the noon angle (LETTER L) and how much it measures (90 degrees). Have the students describe in detail the three kinds of angles: ACUTE (less than an "L"), RIGHT (exactly an "L"), and OBTUSE (more than an "L"). Next, ask the class what an angle whose sides move in opposite directions is called (STRAIGHT). Focus on the 10:00 AM angle once again. Have the class draw the angle that will be formed from 10:00 AM to 6:00 PM. Direct the attention to these two angles and ask what will happen if they are put next to each other and superimpose the common side. Since these two angles form a straight angle of 180 degrees, tell the class that these are SUPPLEMENTARY angles or one is the supplement of the other. Next, using the 10:00 AM angle again, have them draw the angle from this hour to 12:00 noon. As before, put them side by side and superimpose the common side. What angle is formed now (RIGHT ANGLE)? These are called COMPLEMENTARY angles or one is the complement of the other.

DRILL: Show two different angles using the sundial and let volunteers give the corresponding complements and supplements.

MEDIAL SUMMARY: Have the students define the following terms: ACUTE, OBTUSE, RIGHT, SUPPLEMENTARY, and COMPLEMENTARY

APPLICATIONS AND DRILL: If the sun continues to shine up to 6:30 PM, ask the class to draw the angle formed. Ask the class for a name that describes this angle (REFLEX). Continue showing different kinds of angle and have them identify the kind of angle and give the corresponding complements and supplements.

FINAL SUMMARY AND CONCLUSION: Call on a student to describe the different kinds of angles that they have encountered today. Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: The homework is as follows:

I. For each of the following angle measurements, (a) draw and label the angle, (b) describe what kind it is, and (c) draw it on the sundial and tell what time it is, (d) find the corresponding measures of its complement and supplement:

a) 15^{0} b) 45^{0} c) 30^{0} d) 105^{0} e) 135^{0}

II. Define the following terms accordingly:

a) ACUTE b) RIGHT c) OBTUSE
d) SUPPLEMENTARY ANGLES e) COMPLEMENTARY ANGLES.

III. Describe some of the problems associated with using sundials.

SPECIAL EQUIPMENT NEEDED: overhead projector, projection screen, transparencies, markers, pen/pencil and paper

IF TIME: Have the students write what they believe as the beginnings of the sundials. Additionally, describe the problems associated with the sundial in telling the time.

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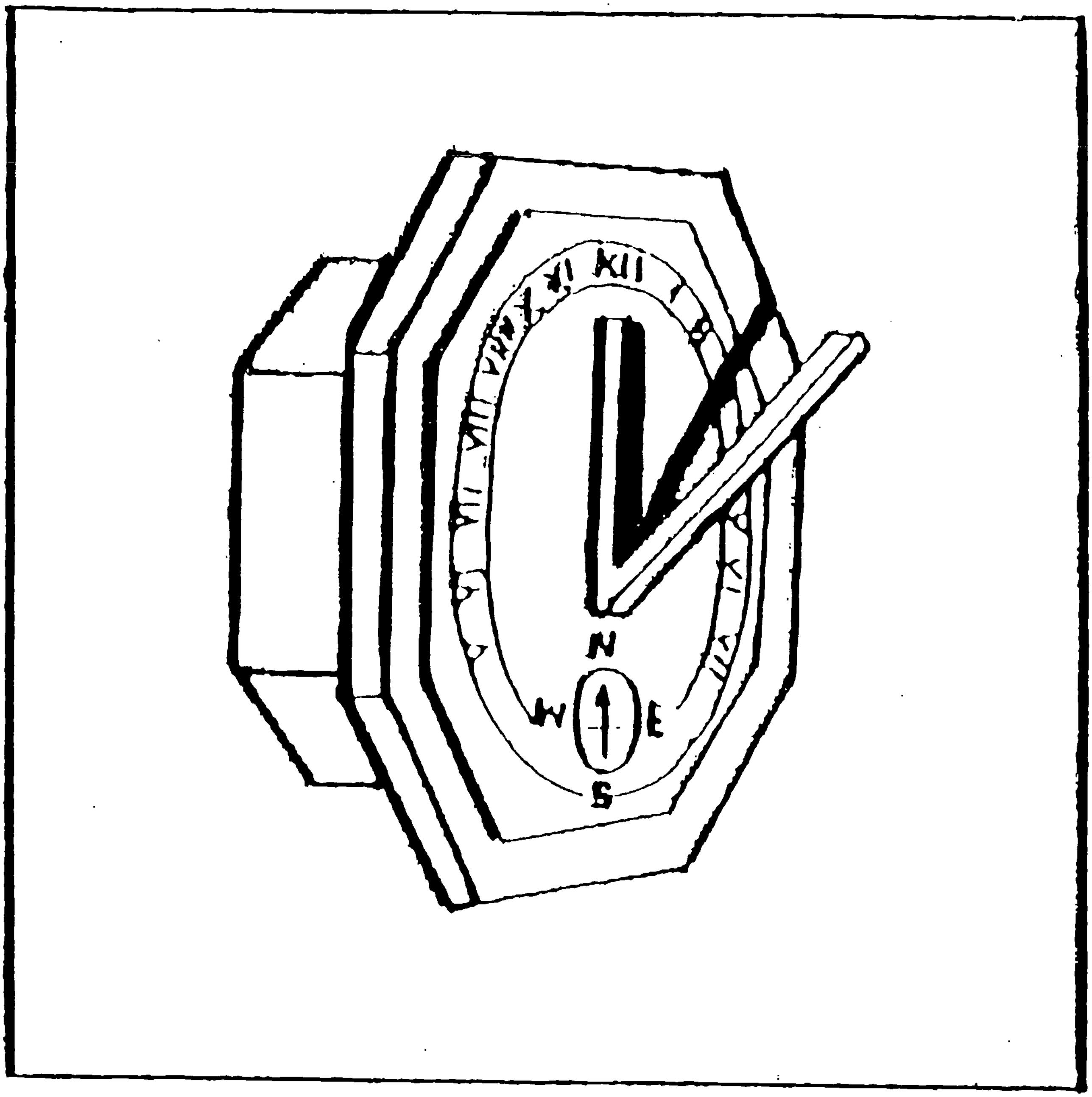
Figure A

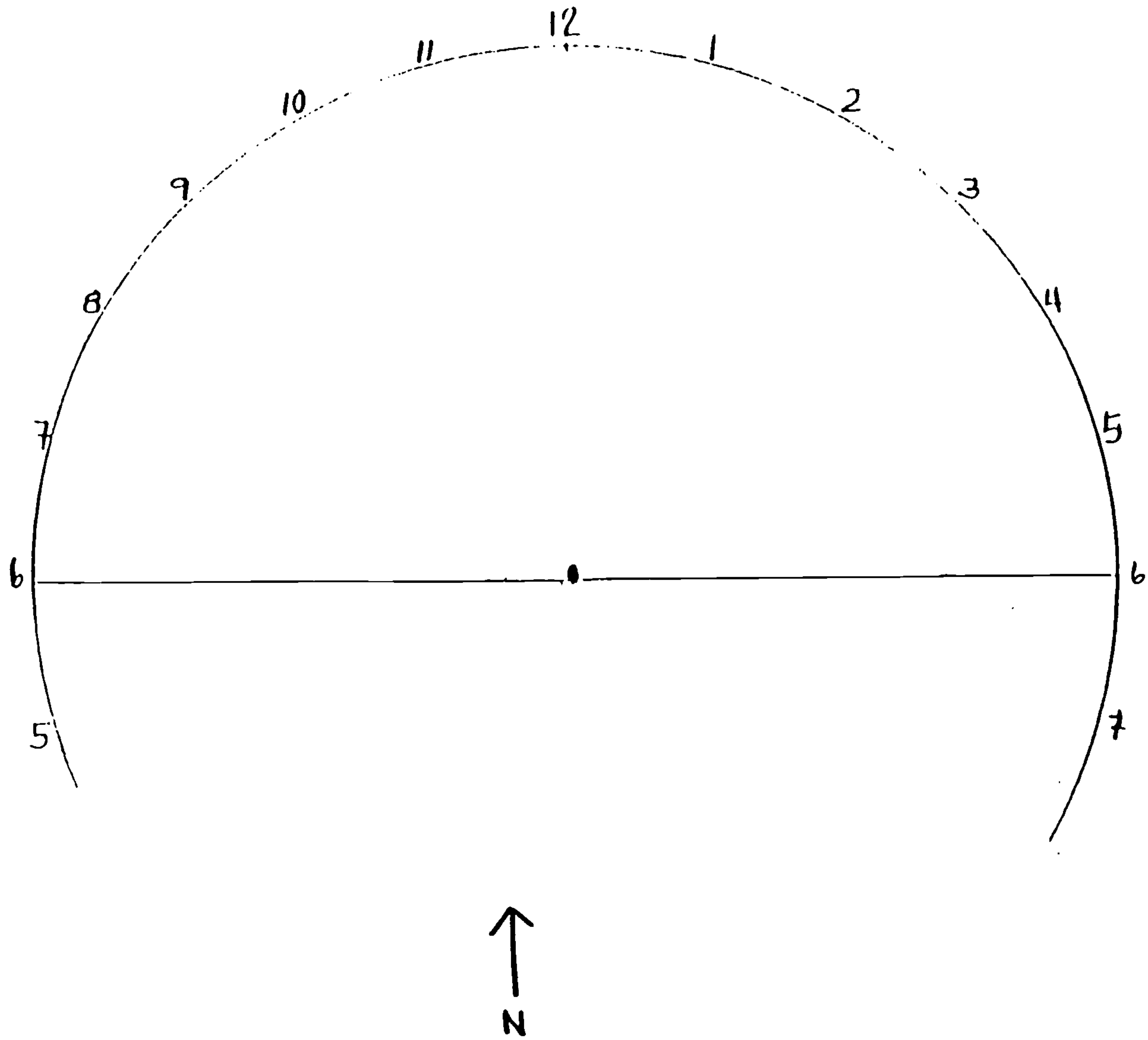
Sundial (1)



Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

Figure B

Sundial (2)



TOPIC: PARALLEL & PERPENDICULAR LINES

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles

AIM: 1) to identify parallel and perpendicular lines, 2) to distinguish the difference between parallel and perpendicular lines, 3) to use Scottish Tartans to identify parallel and perpendicular lines, 4) to draw a Tartan as an application of the lesson

MOTIVATION: Ask the class if they know the song "Old MacDonald." After calling on some volunteers, ask them what they think the name MacDonald means (Son of Donald). Extend it further by saying that "Mac" is of Scottish origin meaning "son of." Additionally, explain to the students that the Scottish clans are known by their TARTANS, which are similar to some families' "Coat of Arms." (This can further be enhanced by showing to the class a piece of cloth with a Tartan pattern.) Mention also that the Tartans carry mottoes with them likewise.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show the transparency that has a sample Tartan pattern. Ask for volunteers to describe what a Tartan looks like. Lead them into saying that the pattern is made of straight lines that "intersect." Point to any two "non-intersecting" lines and ask what they can conclude (1) if these two lines go on and on [They will never meet.] and (2) about the distance between these two lines at any given point [At any given point, the distance between them is the same.]. Explain that these are parallel lines with the properties that they have just described. Next, point to any two intersecting lines and ask what they can say about these them. Once again, lead them into concluding that these lines intersect and are "perpendicular," (They form a right angle.). Ask another volunteer about the intersection of any two lines (a point). Show this case on the transparency.

DRILL: Ask students to give other examples of parallel and perpendicular lines.

MEDIAL SUMMARY: Have a student differentiate between parallel and perpendicular lines.

APPLICATIONS AND DRILL: Ask the class if it possible for two lines to be (1) neither parallel nor perpendicular, (2) neither parallel nor intersecting (Yes on both questions). Have them illustrate these cases. Furthermore, ask them if these special cases mentioned above have names: (1) intersecting lines and (2) skew lines.

FINAL SUMMARY AND CONCLUSION: Call on a student to describe the different cases two lines may have: parallel, intersecting but not perpendicular, intersecting and perpendicular, and skew. Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: The students must draw their own "Tartan pattern" and be able to describe them to the class. Ask them to color these Tartans, give them names, and create their own mottoes likewise.

SPECIAL EQUIPMENT NEEDED: overhead projector, projection screen, transparencies, markers, pen/pencil and paper, color markers or crayons

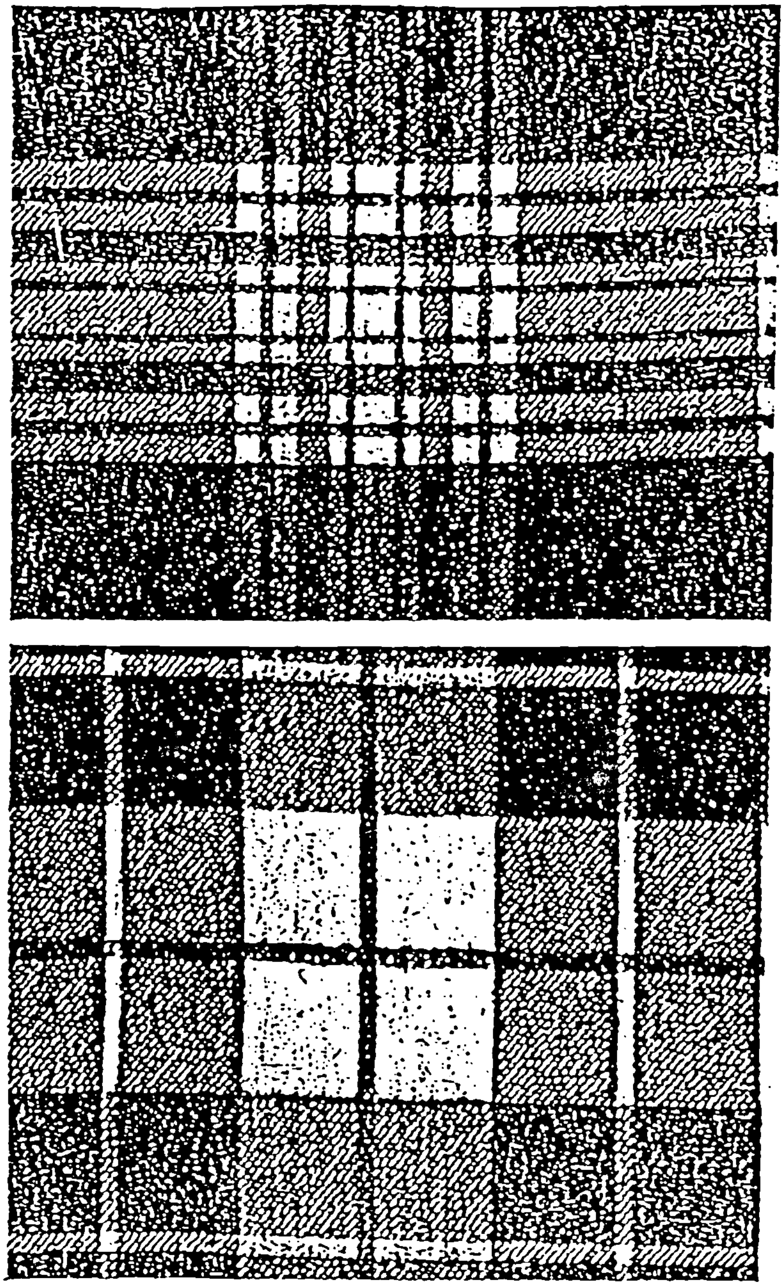
IF TIME: Ask the students what is the difference between "Mac" and "Mc," the former is Scottish and the latter is Irish. Also, have a discussion on the advantages or disadvantages the Tartan has over the Coat of Arms.

REFERENCES:

Scottish Tartans. (1978). London, England: Pitkin Pictorials LTD.

Figure C

Scottish Tartans



Source: Scottish Tartans. (1978). Pitkin Pictorials LTD.

TOPIC: TRANSVERSALS

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles

AIM: 1) to identify parallel lines cut by a transversal, 2) to identify the exterior and interior angles and vertical angles, 3) to use a Zulu belt pattern in learning the concepts, 4) to draw similar patterns

MOTIVATION: Ask the class if they know who SHAKA ZULU was. Tell the class a short history of who he was and what his tribe are all about.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show the transparency that has the Zulu belt pattern. Ask them what figures do they see -- diamonds, lines crossing each other, a zigzag line, etc. Point to any two consecutive lines and ask how these two are related -- parallel. Ask further if these two lines are parallel, what the opposite diagonal line does to them -- it intersects these two lines. The name given to this line is the TRANSVERSAL. Show a separate transparency that has two parallel lines being cut by a transversal. Label the parts. Use this time to discuss the following: exterior and interior angles, alternate interior angles, vertical angles and sum of the measures of the interior angles, etc. Remind the students about the letter "Z" or "N," "F," and "C" to help them remember the theorems.

DRILL: none

MEDIAL SUMMARY: Call on a student to describe what a transversal is. Furthermore, have them recall the newly-introduced concepts.

APPLICATIONS AND DRILL: Have them work on some more examples to demonstrate the principles of parallel lines that are cut by a transversal.

FINAL SUMMARY AND CONCLUSION: Ask the students to state the relationships once again. Give them time to write these relationships into their notebooks. Afterwards, give the class their homework for this lesson.

HOMEWORK ASSIGNMENT: The homework for this lesson is as follows:

1) Draw line AB. Below it, draw line CD parallel to line AB. Line EF intersects line AB and line CD at H and G respectively. Make sure point F is below line CD. If $m\angle AHE = 47^0$, find the measures of the other angles.

2) Using the figure that you just have drawn, draw another transversal KL such that it intersects line AB and line CD at H and J respectively. Make sure that point J is to the right of G. If $m\angle JGF = 120^0$ and $m\angle HJG = 60^0$, find the measures of the other angles.

3) Repeat #2 but this time, make point J to the left of G. What can you conclude?

4) Give examples where you find a design that has parallel lines cut by transversals.

SPECIAL EQUIPMENT NEEDED: overhead projector, projection screen, transparencies, markers, pen/pencil and paper

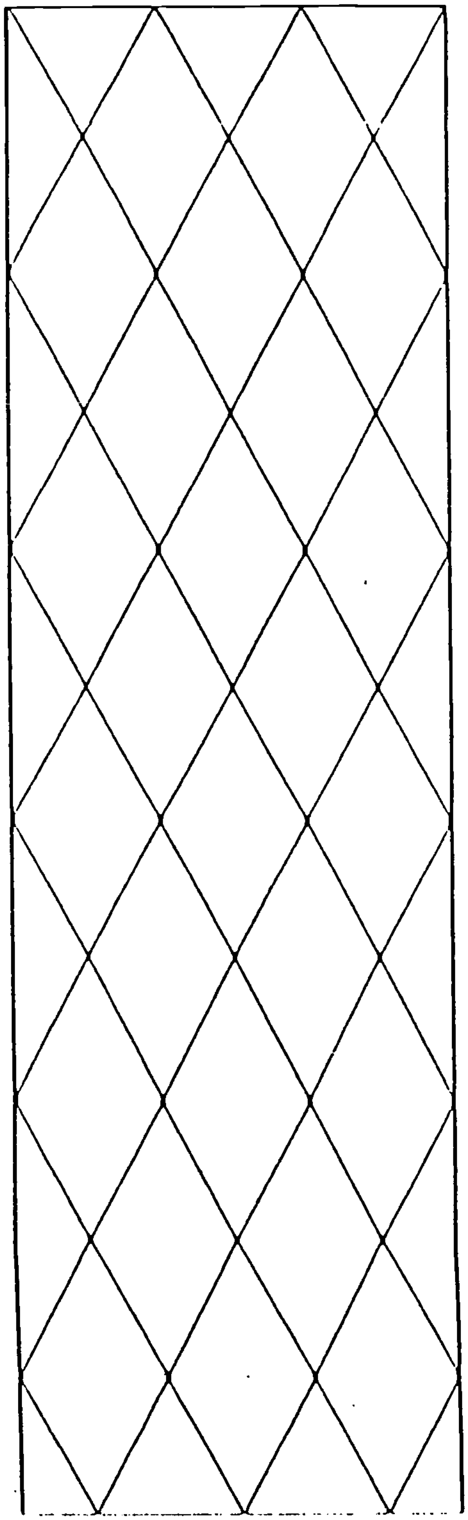
IF TIME: Ask the students why such a word -- TRANSVERSAL. Have them dissect the word and see if they can analyze how the word came about. Additionally, allow them to discuss what they know about the great Zulu tribe of South Africa.

REFERENCES:

Irons, C. and Burnett, J. (1994). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Figure D

Zulu Belt Pattern



Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

TOPIC: TRIANGLES

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles

AIM: 1) to identify the different kinds of triangles, 2) to use Hawaiian petroglyphs in identifying the kinds of triangles, 3) to draw a Hawaiian petroglyph with a short description as an application of the lesson

MOTIVATION: Write the following words and ask what these have in common: hula, lei, luau, poi, beaches, Aloha. Clearly, the words are all about Hawaii. Tell the class that they will learn something about the 50th state, HAWAII.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show the transparency with a picture of a Hawaiian petroglyph and hand each student a copy of the figure. Ask them what figure primarily makes up the petroglyph (triangle). Make it known to them that petroglyphs have been found all over the world where early people lived. Have a volunteer describe what the entire figure is (a man). Tell the students that if a triangular and muscled figure depicts a person, this is a Hawaiian petroglyph and is different from other petroglyphs found elsewhere in the world. Hawaiian petroglyphs are called "*kaha hi ' i* " which means scratched picture. Have them focus their attention to the lower part of the raised arm. Compare, using a ruler, the lengths of the three sides of the triangle and ask them to describe them (The sides do not have the same measurements.) The name for this triangle is SCALENE. Next, they should focus on the upper part of the raised arm. As with the first one, compare the sides. They will arrive to the conclusion that the two sides have the same measurements. The name for this triangle is ISOSCELES. Finally, have then look into the torso of the figure. What can they say about the sides (They all have the same measurements.). The name for this triangle is EQUILATERAL. Conclude that a triangle may be described as scalene, isosceles or equilateral based on the sides. Next, have them focus on the thighs. Using a protractor, have them measure the angles of any of the selected triangles. They should discover that all the angles measure less than 90 degrees or are acute angles. The name for this triangle is ACUTE. Next, measure the angles of the triangles found in the legs. As before, measure the angles. Each triangle has one right angle and two acute angle. The name for this triangle is RIGHT. Finally, ask them to measure the angles of the triangles of the lower part of the lowered arm. They will realize that there is one obtuse angle and two acute angles; thus the name of this triangle is OBTUSE. Conclude that a triangle may be classified also according to the angles.

DRILL: Ask students to sum the measurements of the angles in each triangle (180 degrees).

MEDIAL SUMMARY: Have a student state the six classifications of the triangle.

APPLICATIONS AND DRILL: Ask the class to measure the angles of the triangle in the torso. Being an equilateral triangle, the angles measure 60 degrees each. What conclusion can they formulate? (All the angles in an equilateral triangle measure the same; hence, EQUIANGULAR.)

FINAL SUMMARY AND CONCLUSION: Call on a student to summarize the different classifications of the triangle and to state the sum of the measures of the angles of a triangle. Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: The students must draw their own "Hawaiian Petroglyph" and be able to describe them to the class. Remind them to use all the different kinds of triangles in their figures. They may give it a name and write a short story/description of this petroglyph.

SPECIAL EQUIPMENT NEEDED: overhead projector, projection screen, transparencies, markers, pen/pencil and paper, handout, ruler, protractor

IF TIME: Mention that Hawaiian petroglyphs, in addition to triangles, have circles, concentric circles, dots connected to curving lines, and U shapes. Start a discussion on why early people carved such designs on a rock. Furthermore, discuss what other things are native to the Hawaiian archipelago.

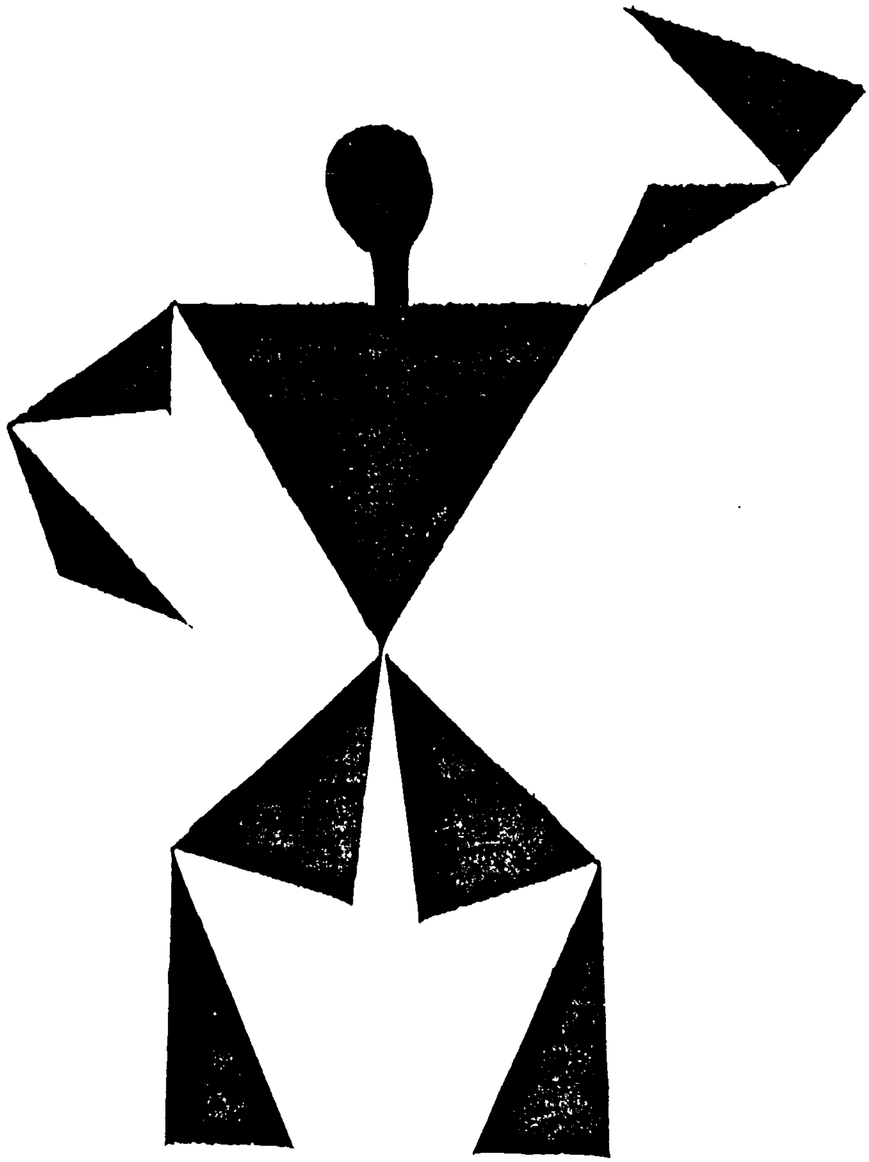
REFERENCES:

Irons, C. and Burnett, J. (1994). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Krause, M. C. (1983). Multicultural Mathematics Materials. Reston, VA: NCTM Publications.

Figure E

Hawaiian Petroglyph



Source: Krause, M. (1983). Multicultural Mathematics Materials.

TOPIC: QUADRILATERALS

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, parallel lines, angles

AIM: 1) to identify the different kinds of quadrilaterals, 2) to use the Hopi Bird design in learning the concepts, 3) to draw a Hopi Bird design to apply the learned concepts

MOTIVATION: Ask the class to give examples of birds. Ask them what characteristics are special to these birds. Then, tell them that birds are often seen and used in designing different items like pottery, paintings, etc.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show the class the transparency that has the Hopi Bird design. Ask the class what shapes they see. After hearing a few answers, instruct the class to focus on the figure underneath the head in the body of the bird. Ask how many sides there are to this shape (4). Next, ask what this figure is generally called (quadrilateral). Explain that this, literally translated, is *four sides* . Have them locate other quadrilaterals. Then, have them recall how triangles were classified (according to sides and angles). State that quadrilaterals may also be classified accordingly. Have them look at the second quadrilateral from the bottom. Have them describe it-- 2 parallel sides, 2 non-parallel sides. If these conditions are satisfied, instruct them that this is called a TRAPEZOID. Ask them to look for more trapezoids. Next, connect the two wingtips by drawing a straight line. Draw a straight line passing through the "waist" of the bird. Have them look at the quadrilateral that was formed on the left side. Have a volunteer describe the shape -- 2 pairs of parallel sides. the opposite sides are of the same length, etc. If these conditions are satisfied, this figure is called a PARALLELOGRAM. As before, ask them to look for more parallelograms. Before proceeding, have the students differentiate between a trapezoid and a parallelogram. Next, lead them to the figure in the center of the bird. Ask if this is a trapezoid or a parallelogram. Have them describe the angles in the figure (All are right angles.). State that this is known as a

RECTANGLE. Ask for a definition of a rectangle-- a parallelogram whose angles are all right angles. Then, have them focus again on the figure on the second row from the bottom. Measure how long the left side is and mark the same distance on the parallel sides. Next, connect these two points. Have them describe the newly created figure -- all sides are of the same length, opposite sides are parallel. State that this is known as RHOMBUS. Have them differentiate between a rectangle and a rhombus. Finally, ask the question if there is such a figure that is a rectangle and a rhombus at the same time (SQUARE). Then, ask for a definition of a square -- a parallelogram whose sides are of the same length and whose angles are all right angles.

DRILL: Have them locate the different quadrilaterals in the design.

MEDIAL SUMMARY: Have them define the different kinds of quadrilaterals.

APPLICATIONS AND DRILL: Give the class some time to count how many trapezoids, parallelograms, rectangles, rhombuses, and squares are in the figure.

FINAL SUMMARY AND CONCLUSION: Read the following to the class: "The Hopi Indians of northeastern Arizona use geometric shapes and designs to decorate their beautiful pottery ware. Hopi potters use the coil method to make their pottery. Ropes of clay are spiraled, one on top of the another. The pot is smoothed, dried, and coated with watery clay. Then the pot is polished with a smooth stone. The design is carved or painted on the pot before firing." Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: For their homework, have the students draw their own bird designs that utilize triangles and quadrilaterals. They may color the designs. Additionally, write a short description about the design.

SPECIAL EQUIPMENT NEEDED: transparency, overhead projector, projection screen, pen/pencil and paper, crayons or color markers

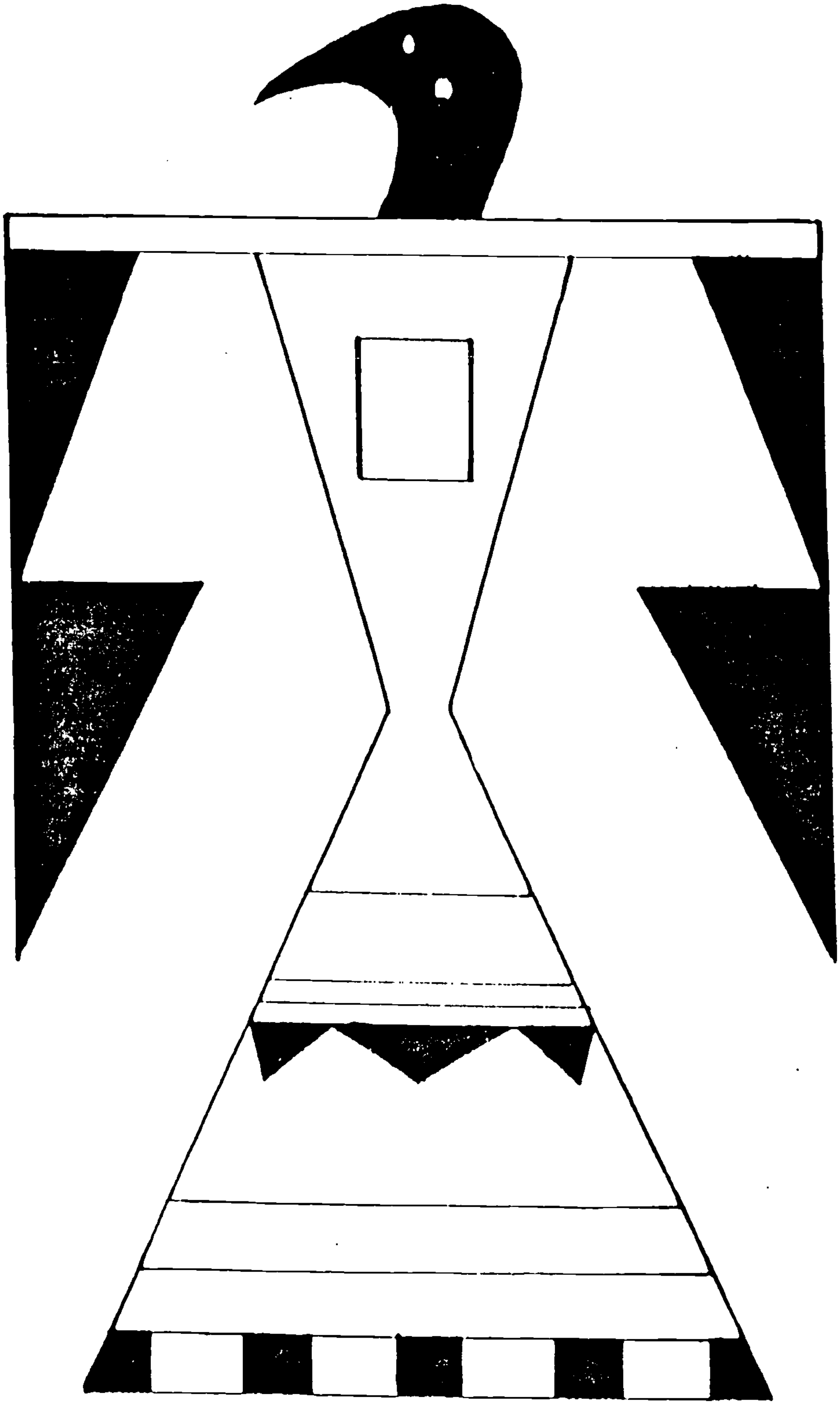
IF TIME: Start a discussion on where else, they think, they can find the design other than in pottery and what other designs there may be.

REFERENCES:

Krause, M. C. (1983). Multicultural Mathematics Materials. Reston, VA: NCTM Publications.

Figure F

Hopi Bird Design



Source: Krause, M. (1983). Multicultural Mathematics Materials.

TOPIC: POLYGONS

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, quadrilaterals

AIM: 1) to identify names of different polygons, 2) to use TANGRAMS to form different polygons, 3) to use the Tangram pieces to create different figures

MOTIVATION: Ask the students what their favorite games or puzzles are. After hearing some games, survey who among them have heard of TANGRAMS. Say that these Tangram pieces will be used to explore other geometric figures.

DO-NOW EXERCISE: Give each student a handout that contains the seven pieces of the Tangram. If this does pose a problem due to time constraint, this may be done ahead of time.

DEVELOPMENT AND METHODS: Instruct the class to place the Tangram pieces on their desks. Then, ask them to take any two unlike pieces and connect them anywhere they want. On a piece of paper, have them trace the outline of the figure that they have just created. After doing this, have them take another piece of the Tangram and connect these three pieces together. As before, trace the outline onto the paper. Set the pieces aside and focus on the outline of the figure. Ask how many sides the first figures have and tally the results. Do the same thing with the second figures. Ask what name is generally given to a four-sided figure (quadrilateral). For five-sided to ten-sided, ask the students who can give the corresponding names -- PENTAGON, HEXAGON, HEPTAGON /SEPTAGON, OCTAGON, NONAGON, and DECAGON respectively. Have them decode the pattern based on the prefixes used. Next, ask them to look at the square piece. State that this is called a REGULAR polygon. Brainstorm the class as to what REGULAR means -- all sides are of the same length and all angles measure the same.

DRILL: Ask what a regular triangle is called.

MEDIAL SUMMARY: Have them define what regular means and the names for the different polygons.

APPLICATIONS AND DRILL: Have a contest on who can form the polygon that has the most sides using all the seven pieces of the Tangram.

FINAL SUMMARY AND CONCLUSION: Before the homework is given, read the following to the class: "The Tangram is about 4000 years old and has a long history. It is believed that tangrams were named after Tan, a legendary Chinese scholar. Legend has it that a porcelain tile was dropped and broke into 7 pieces. As he tried to reconstruct the square, he created various shapes." Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: Using the Tangram pieces, have them create at least five real-life figures. Make sure that these figures are related and have them write a short story about these figures. Also, they are to indicate the numbers of sides of each figure.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, handout with Tangram illustration, scissors

IF TIME: Since there is a contest on who can come up with the polygon with the most sides, this activity will utilize the entire period.

REFERENCES:

Burton, G. M., et al (1994). Mathematics Plus. Orlando, FL: Harcourt Brace and Company.

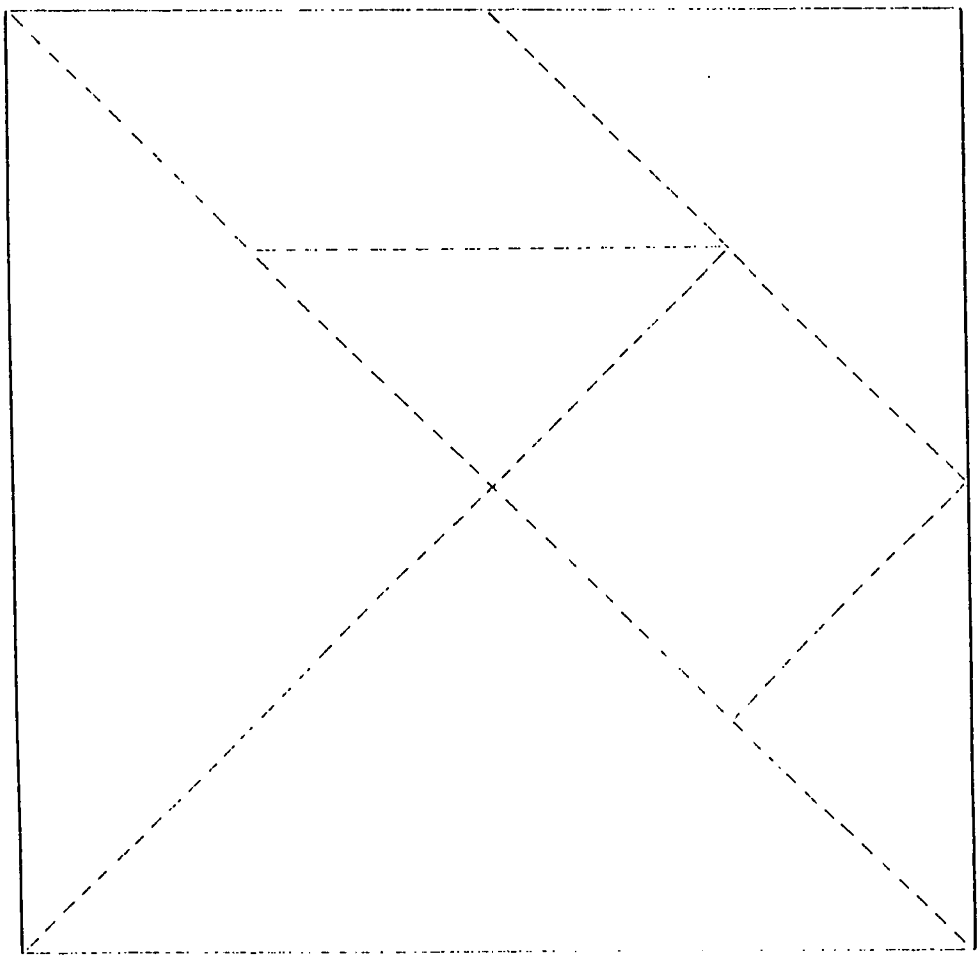
Croom, L., et al (1992) Mathematics, Exploring Your World, Exploring your Multicultural World Mathematics Projects. Morristown, NJ: Silver Burdett Ginn Inc.

Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Multicultural Mathematics Posters and Activities (1984). Reston, VA: NCTM Publications.

Figure G

Tangram



Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

TOPIC: CIRCLES

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines

AIM: 1) to identify the parts of a circle, 2) to use Medicine Wheel to name the different parts of a circle

MOTIVATION: Call on volunteers to ask for their most important dates of the year. Ask how they remind themselves of these dates. State that in the early days, people used a different method of knowing what time of the year it is. Some people used what is called a "Medicine Wheel." State further that this is a device to tell time and the early white pioneers coined the term "medicine wheel" due to its similarity to wagon wheels.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show them a transparency of a medicine wheel. Tell them that medicine wheels were used by Native American Indians to study stars and mark important dates like the summer solstice. Medicine wheels are made up of a large circle of stones. Focus their attention to the wheel. Ask them what they can say about the line segments-- almost identical, will meet in a point eventually, etc. Now, ask them what is the name of this point (center). Next, ask them what is the name of the line segment from the center to the circle (radius). Then, try to connect two "radii" that will form a straight line. Ask what name is given to this part of the circle (diameter). Given this hint, what can be said about the relationship between a radius and a diameter -- diameter is twice the radius. Then, point to any two consecutive radii and ask what the section bounded by the two radii and part of the circle is called (sector). Then, what do we call a portion of the circle (arc). Repeat the newly introduced terms. Next, connect the endpoints of two non-adjacent radii. Ask the class what this is called (chord). Ask for a definition of a chord of a circle. How is the chord related to the diameter -- the longest chord is the diameter. Ask next how many degrees are in a circle (360). How is this justified? Engage the class into sharing of ideas on how the circle has 360 degrees.

DRILL: Ask a volunteer to answer what the diameter is if the radius is 4.

MEDIAL SUMMARY: Have the students define in their own the different parts of the circle..

APPLICATIONS AND DRILL: Redirect their attention to the medicine wheel. Ask how many radii will there be if we want the angle formed by any two adjacent radii is 30 degrees. Extend it by asking different degrees. Reverse the question by giving the number of radii and ask for the degree of the angle formed.

FINAL SUMMARY AND CONCLUSION: Read the following to the class: "One famous medicine wheel is the BIG HORN MEDICINE WHEEL in Wyoming. it was built by the Crow, a Native American tribe from the Northern Plains. This wheel is used by the Crow natives to identify the summer solstice and track the paths of various stars and is approximately 92 to 95 feet across. The inner circle is close to 12 feet across. Furthermore, it is composed entirely of stones. Each spoke of the wheel is made up of anywhere from 20 to 50 rocks, and there are 28 spokes in all." Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: Make sure they know the measurement across of The Big Horn Medicine Wheel. They are to create their own medicine wheels with different radii; however, the ratio of the radii of their medicine wheel should EQUAL the ratio of the Big Horn Medicine Wheel! Remind them that they have to use "stones" in creating the spokes and the wheel. Be prepared to explain their work.

SPECIAL EQUIPMENT NEEDED: transparency, overhead projector, projector screen, pen/pencil and paper, ruler

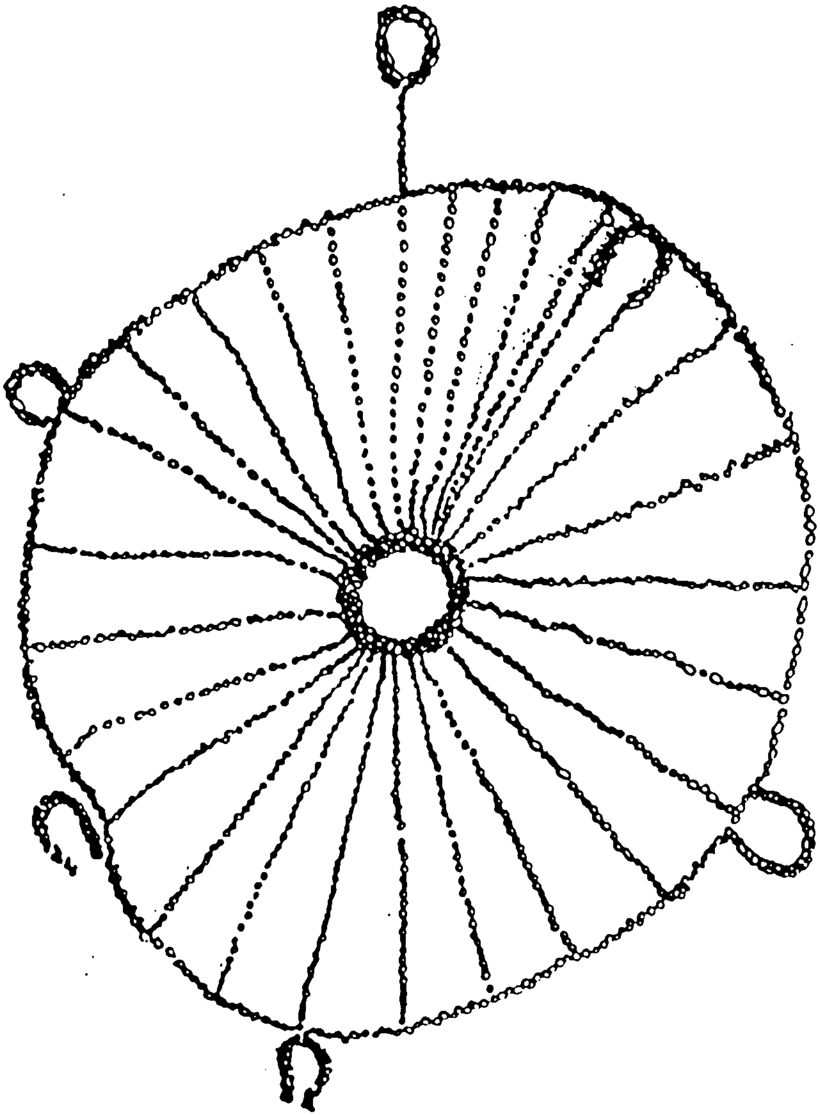
IF TIME: Engage them in a discussion on whether or not this device is accurate. If not, what could be the possible source(s) of errors.

REFERENCES:

Croom, L., et al (1992) Mathematics, Exploring Your World, Exploring your Multicultural World Mathematics Projects. Morristown, NJ: Silver Burdett Ginn Inc.

Figure H

Medicine Wheel



Source: Croom, L. et al. (1992). Exploring Your Multicultural World Mathematics Projects.

TOPIC: RIGHT TRIANGLES

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles

AIM: 1) to identify the parts of a right triangle, 2) to use the method of the Egyptian Rope Stretchers in proving the "Pythagorean Theorem", 3) to apply the Pythagorean Theorem in determining the lengths of the sides of a right triangle

MOTIVATION: Stretch a yard of cotton twine and show it to the class. Ask for some uses of the twine. Mention to the class that this "surveying" instrument was used once by the Ancient Egyptians.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Group the class into three's and give each group a piece of cotton twine. Have the group make equally-spaced knots. 6 inches between knots would be favorable. Each group should have at least thirty knots. When they have finished, ask them to count to 12 knots. Using only these twelve knots, have each group form a right triangle. Ask for the number of knots per side (3, 4, 5). Focus their attention to the side with 5 knots. Ask a volunteer to describe this side--the longest side of the right triangle, directly opposite the right angle, etc. Ask for a name for this side (HYPOTENUSE). Next, ask them to square each of the dimensions and try to figure the relationship that can be concluded. Give them time to come up with the principle that in a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse. Mention to the class that this is known as the Pythagorean Theorem. Also, tell the class that the Greek Pythagoras was not the only person in the ancient world to have discovered and used this relationship. Next, direct the class to the dimensions again. These numbers, collectively, are known as <u>Pythagorean Triples</u>.

DRILL: Have them draw a right triangle of dimensions 3 in, 4 in, and 5 inches.

MEDIAL SUMMARY: Have a student state the Pythagorean Theorem.

APPLICATIONS AND DRILL: Give the class some more time to look for other Pythagorean Triples. Assist those who might require help and further clarification.

FINAL SUMMARY AND CONCLUSION: Read the following to the class: "Even though it is known that as early as 2000 BC, the Egyptians are aware that $4^2 + 3^2 = 5^2$, there is no evidence that the Egyptians knew or could prove the Right Triangle Property. Ancient Egyptian tombs have pictures of scribes and their assistants carrying ropes with equally spaced knots on them. Furthermore, it has been suggested that these rope stretchers, the HARPEDONAPTAI, helped reestablish land boundaries after the yearly flooding of the Nile. It has also been suggested that rope stretchers helped in the construction of the pyramids." Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: The homework for this topic is as follows:

I. For each of the set of numbers below, find the third number to make them Pythagorean Triples. Additionally, draw the corresponding right triangle. Label the sides and find the measures of the angles.

a) 8, 15, ? b) ?, 12, 13 c) 6, 8, ? d) 15, 9, ? e) 20,12, ?

II. Is it possible to draw an isosceles right triangle? If yes, draw it. If not, justify why not. What can you conclude about the other two acute angles of the triangle?

III. Write a general statement about the relationship of the acute angles in a right triangle.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, ruler, cotton twine

IF TIME: Have them discuss whether or not it is fair to call this principle "Pythagorean Theorem" given the fact he did not discover it.

REFERENCES:

Baumgart, J. K., et al. (1989). Historical Topics for the Mathematics Classroom. Reston, VA: NCTM Publications.

Eves, H. (1990). An Introduction to the History of Mathematics with Cultural Connections. Philadelphia, PA: Saunders College Publishing.

Li, Y. and Du, S. R. (1987). Chinese Mathematics, A Concise History. Oxford, England: Clarendon Press.

Serra, M. (1993). Discovering Geometry, An Inductive Approach. Berkeley, CA: Key Curriculum Press.

TOPIC: PERIMETERS & AREAS OF POLYGONS

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, quadrilaterals, polygons

AIM: 1) to establish the perimeters and areas of polygons, 2) to use a figure in a VEDIC SACRIFICIAL ALTAR design to name the polygons , 3) to use the formulas to solve for perimeters and areas of polygons

MOTIVATION: Call on a student and ask what a FALCON is. Ask for descriptions of the falcon and also ask for any other birds of prey that the class might know. Tell them that a falcon may be drawn using only polygons.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show to the class an illustration of Vedic Sacrificial Altar. Ask them what is the design of the figure. Have them list the different kinds of polygons that they see. Ask them what to do if one wishes to find out the "distance around the figure." Lead the class into saying that the lengths of the sides are added to find this distance. Ask the class if there is a name for this distance --PERIMETER. Reiterate to the class that the perimeter of a polygon is adding the lengths of the sides. Next, ask the class this question: "If the design were to be placed on the floor, how much of the floor will be covered?" Brainstorm with the class for ideas. Furthermore, ask the class what essentially are we solving: AREA. Once again, lead them into saying: "Find how much each small section covers and add them up." Having said that, point out that there are different figures involved and each figure will have a different area. This is the best time to give them the formulas for the polygons: triangle = (B x H)/2; rectangle = L x W; square = s^2; parallelogram = B x H; trapezoid = $[(B_1+B_2) \times H]/2$. Make sure that these formulas are known to them before ending the lesson.

DRILL: Ask the class to solve for the perimeter and area of a 9" by 12" rectangle.

MEDIAL SUMMARY: Have the students restate the formulas for the perimeters and areas of the polygons.

APPLICATIONS AND DRILL: Give the class some complicated polygons to solve for the area. Make it a point that these polygons can be sectioned into more familiar polygons.

FINAL SUMMARY AND CONCLUSION: Make sure that the formulas are copied into their notebooks. Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: Have them draw any other animal using only triangles and quadrilaterals. They are to solve for the perimeters and the areas of each type of triangles and quadrilaterals used and of the entire figure. Be prepared to share the drawings with the class.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, projection screen, overhead projector, markers

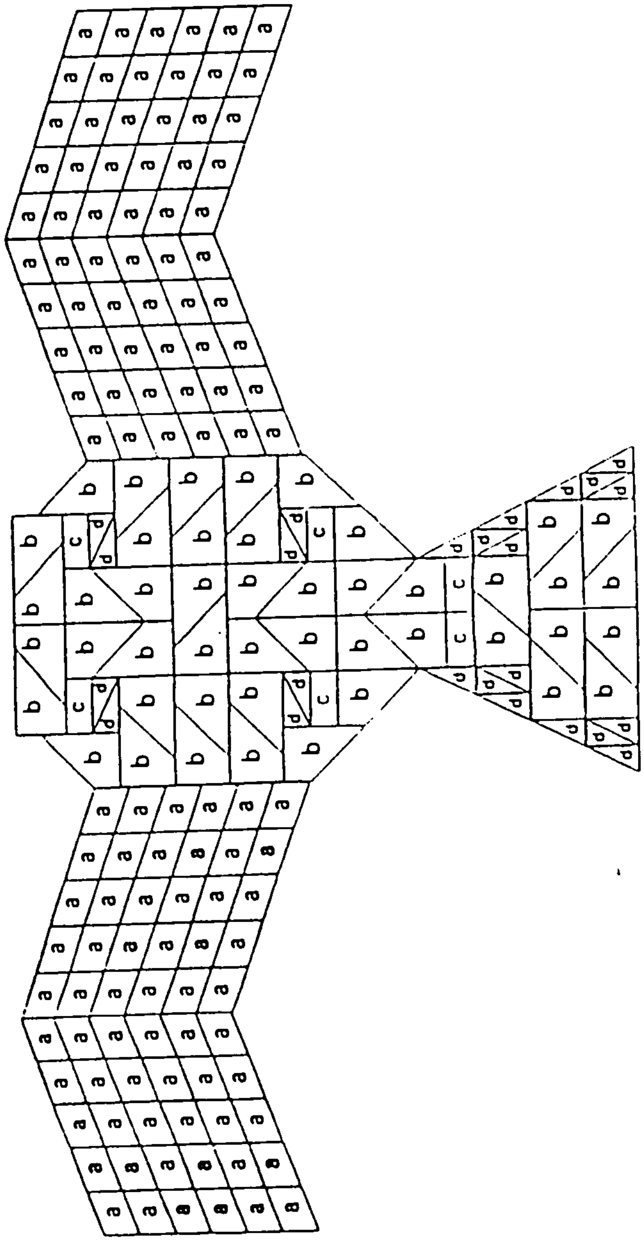
IF TIME: Engage the class in discussing the possible significance of the falcon in the VEDIC SACRIFICIAL ALTAR. Ask also what other kinds of animals could have been used.

REFERENCES:

Joseph, G. G. (1991). The Crest of the Peacock Non-European Roots of Mathematics. London, England: Penguin Books.

Figure I

Vedic Design



Source: Joseph, G. (1991). The Crest of the Peacock.

TOPIC: CIRCUMFERENCE & AREA OF CIRCLES

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, radius, diameter

AIM: 1) to use the formulas to solve for the circumference and area of a circle, 2) to define "π" 3) to use the method of ERATOSTHENES in solving for the circumference of a circle, 4) to draw a MANDALA as an application of the lesson

MOTIVATION: Ask if anyone knows the diameter or the radius of the earth. Call on volunteers to suggest what instruments might be used to measure the diameter or radius of the earth. Explain that there was once a mathematician named ERATOSTHENES who used no complicated instruments but instead used simple geometry principles.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Illustrate to the class how Eratosthenes solved for the circumference of the earth. After doing this, say that there is a formula that solves for the circumference of a circle namely, $C = \pi d$. Use this opportunity to ask the class about π. Mention to them that there have been a lot of values assigned to this number. For the sake of convenience, we will use 3.14 for approximating π. Show an example using the formula. Then, ask them how much will a circle cover. This is also the opportunity to recall what area means and how it was solved for in polygons. For circles, the formula is $A = \pi r^2$. Make sure that these formulas for the circumference and area of the circle are noted.

DRILL: Give a radius and have them solve for the circumference and area of the circle.

MEDIAL SUMMARY: none

APPLICATIONS AND DRILL: Ask the class to solve for the circumference and area of the circles created by the following: a) the length of the right hand as the radius, b) the length of the right hand as the diameter, c) the length of the right leg as the diameter,

d) the distance from shoulder to shoulder as the radius, and e) the distance from left middle fingertip to the right middle fingertip as half the radius.

FINAL SUMMARY AND CONCLUSION: Read the following to the class: "Eratosthenes truly demonstrated what the word geometry really means and made us realize that geometry is all around us." Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: Give the class their handouts for the homework. The instructions are indicated in the handout.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, ruler or tape measure, overhead projector, transparencies, projection screen

IF TIME: Discuss the possible flaws that Eratosthenes had encountered in solving for the circumference of the earth. Suggest alternative ways in solving for the radius or diameter of the earth.

REFERENCES:

Multiculturalism in Mathematics, Science, and Technology: Readings & Activities. (1993). New York, NY: Addison-Wesley Publishing Company, Inc.

Serra, M. (1993). Discovering Geometry, An Inductive Approach. Berkeley, CA: Key Curriculum Press.

Figure J

Mandala Homework

A MANDALA is a design made of concentric circles. The word is of Hindu Sanskrit origin which means circle or center. This is used by the Hindus for meditation. However, mandalas are found in many parts of the world. The Aztecs of Mexico developed a mandala for their calendar. An illustration is found below.

Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

Directions:

I. Create your own MANDALA subject to the following rules:
   a) The radii of the circles are doubling each time.
   b) Draw a maximum of 5 concentric circles.

II. Answer the following.
   a) Find the circumference of each circle.
   b) What is the difference between the circumferences of any two adjacent circles?
   c) Find the area of each circle.
   d) What is the difference between the areas of any two adjacent circles?
   e) Write a statement on how the circumference and the area of a circle change when the radius is doubled. Can you extend this statement when we triple the radius?

III. Design and color the innermost circle and the sections between <u>the 2nd and the 3rd circles</u> and <u>the 4th and the 5th circles</u>. Write a short description of your design. What is the area of the section without the design?

TOPIC: SYMMETRY

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines

AIM: 1) to differentiate between line (bilateral) symmetry and turn (rotational) symmetry, 2) to use the basket designs created by the Hopi Indians, 3) to create designs having different orders of bilateral and rotational symmetries

MOTIVATION: Ask the students in what supermarket do they shop. After calling several students to respond, ask next what they use to carry the few items when shopping. Tell them that just like now, people, like the Indians, use baskets to hold and carry items.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show the transparency with the Hopi Basket design to the students. Have them describe the figure. After a while, ask them to draw a diameter that will divide the design into two matching parts, making one the mirror image of the other. In this figure, there are three bilateral symmetries. The number of bilateral symmetries a figure has is called its order of bilateral symmetry. Next, hold the transparency still by placing a fingertip or a pen/pencil on the center of the design. Let them remain focused as the figure is turned until the original design is seen. Ask them to count how many times, beginning and stopping at the original position, can the figure be turned and get the same design? (3). Explain that this symmetry is rotational symmetry and the number of rotational symmetries is called, once again, order of rotational symmetry.

DRILL: Show another Hopi Basket design and state what are the orders of bilateral and rotational symmetry.

MEDIAL SUMMARY: State the two kinds of symmetry once again.

APPLICATIONS AND DRILL: Show the remaining Hopi Basket designs and have them count the orders of both bilateral and rotational symmetry. Furthermore, let the students determine the orders of bilateral and rotational symmetry of the letters of the

English alphabet written in the their upper case form.

FINAL SUMMARY AND CONCLUSION: Ask for volunteers to define the following: bilateral symmetry, rotational symmetry, order of symmetry. Furthermore, the order of symmetries may or may not be the same. Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: Have the class design their own Hopi baskets having both bilateral symmetry and rotational symmetry of order 5, 6, 8, or 10. Choose only one order.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, ruler, compass, markers, overhead projector, transparencies, projection screen

IF TIME: Discuss the possible reasons why the Hopi Indians used and use such involved designs for their baskets. Additionally, have them analyze the bilateral and rotational symmetries of the Mexican designs.

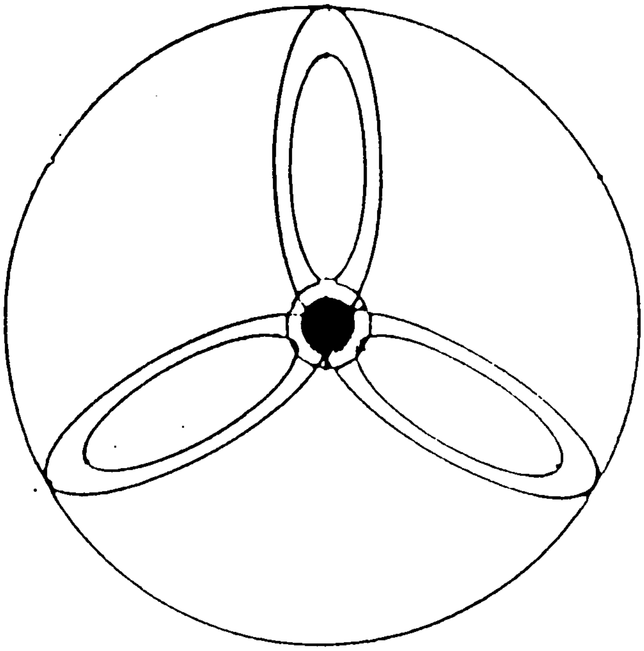
REFERENCES:

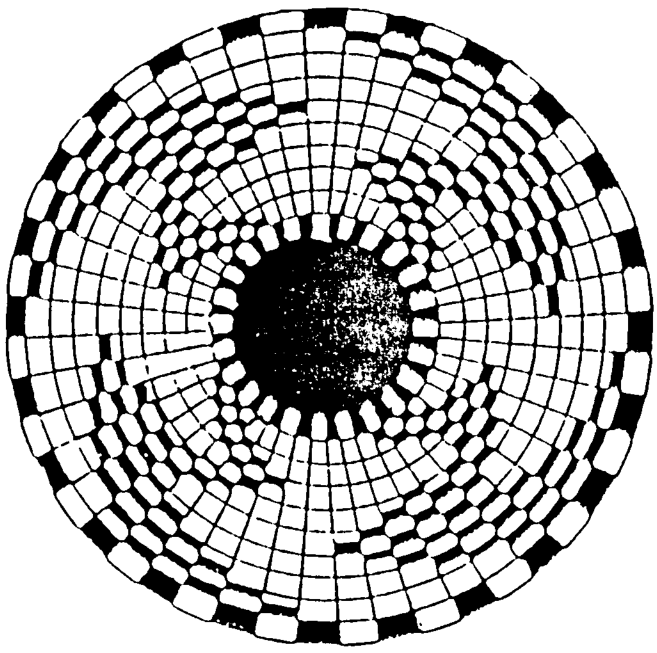
Irons, C. and Burnett, J. (1994). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Zaslavsky, C. (1993). Multicultural Mathematics: Interdisciplinary, Cooperative-Learning Activities. Portland, ME: J. Weston Walch, Publisher.

Figure K

Hopi Basket Designs

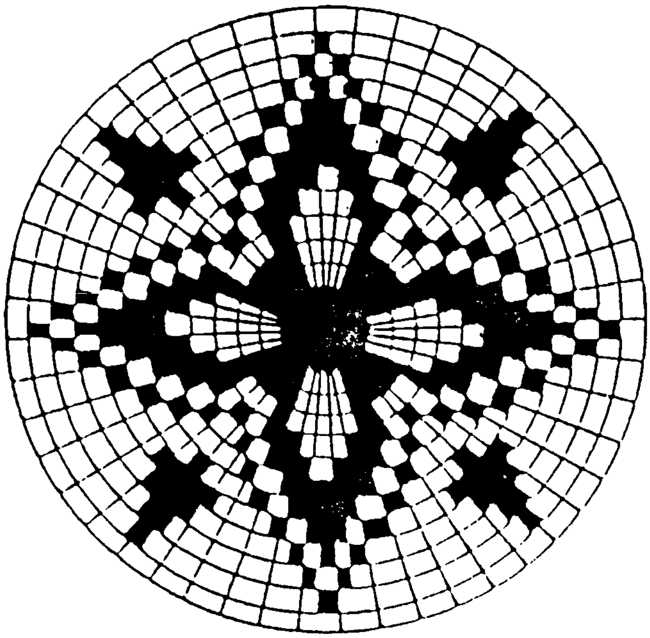


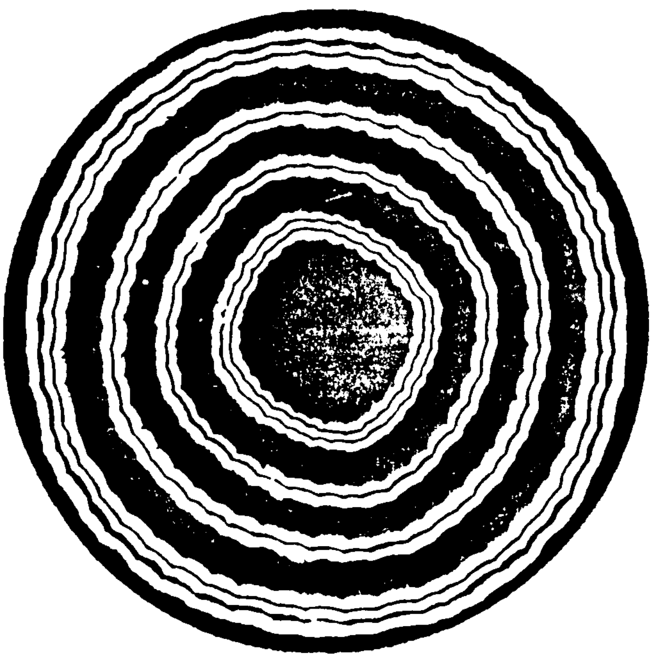


Source: Zaslavsky, C. (1993). Multicultural Mathematics.

Figure K (cont'd.)

Hopi Basket Designs

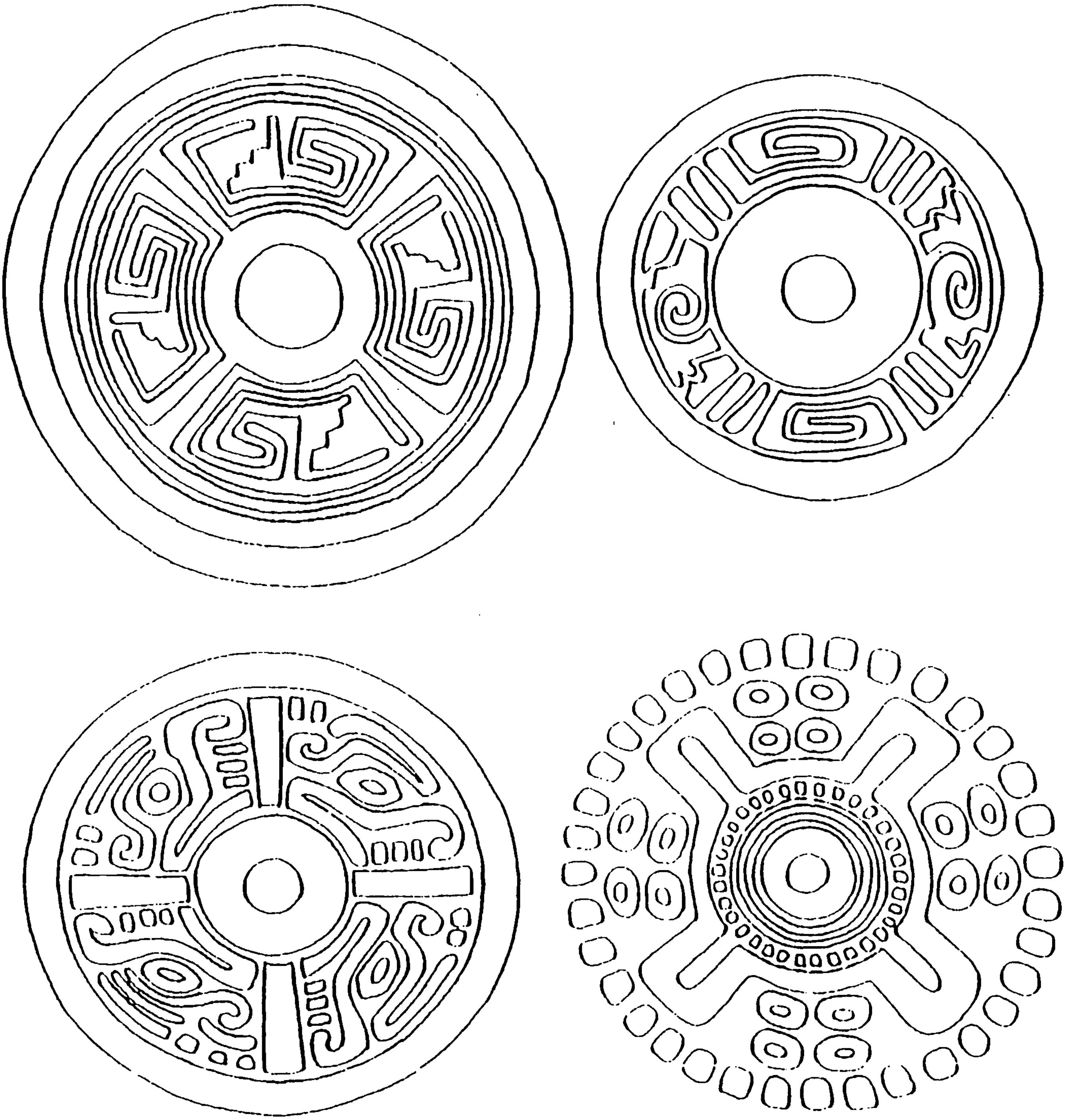




Source: Zaslavsky, C. (1993). Multicultural Mathematics.

Figure L

Designs of Mexico



Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

TOPIC: CONGRUENCE & SIMILARITY

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, polygons

AIM: 1) to differentiate between congruence and similarity, 2) to use a design from the Maoris of New Zealand in understanding congruence and similarity, 3) to use the symbols for congruence and similarity, 4) to copy a design to demonstrate congruence and similarity

MOTIVATION: Ask the class if anyone knows the Maori people. After hearing from the students, tell them a little history of this group from New Zealand. Say next that a design from the Maoris will be used in the following lesson.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show the transparency that has the Maori design. Ask the class to describe the design that they see. What figures make up the design? Point to the second quadrant of the design. Ask for any two designs that are completely alike. Explain that these two figures are congruent, which means having the same size and shape. Ask for any other congruent figures. Additionally, show the symbol " = " which is used for congruence. Next, have them focus on the first quadrant. Ask for two figures that are almost congruent. Have them describe these two figures in detail. Explain that these two figures are similar, which means having the same shape but not necessarily the same size. Ask for another pair of similar figures. Finally, show the symbol " ~ " that is used for similarity.

DRILL: none

MEDIAL SUMMARY: Repeat what congruence and similarity mean.

APPLICATIONS AND DRILL: Explain to the class that a very good application of similarity is scale drawing. The ratio of similarity is the scale given for a drawing. Examples will be in a map. Show, by using grids, how a design may be reduced or enlarged. The final figure is similar to the original figure. since it has the same shape but not the same size. Have them copy a design by both enlarging and reducing it.

FINAL SUMMARY AND CONCLUSION: Have them write the difference between congruence and similarity. Afterwards, give them their homework.

HOMEWORK ASSIGNMENT: Give the class the handout. The instructions are written in the handout.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, grid or graph paper, overhead projector, transparencies, projection screen

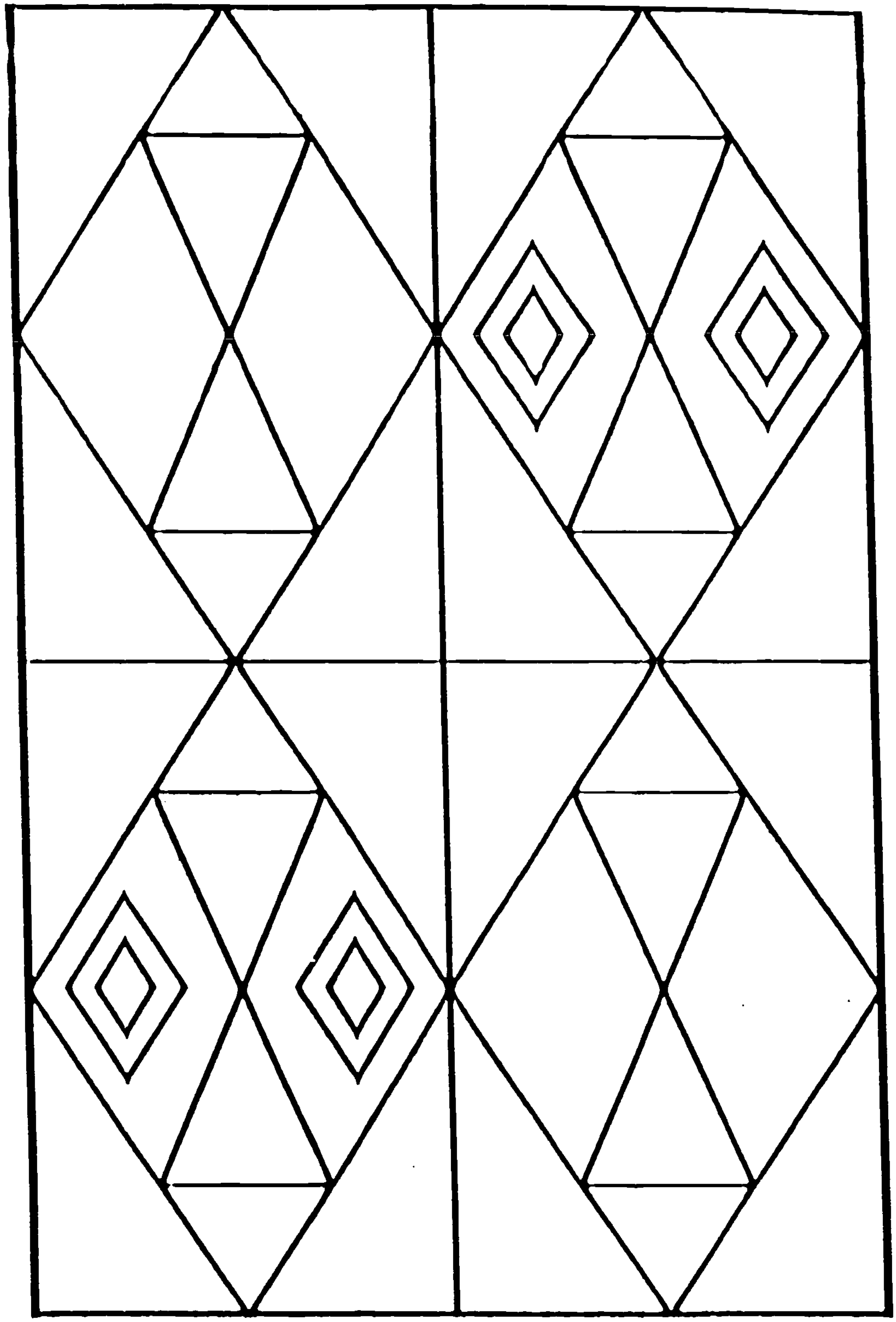
IF TIME: Engage them in discussing which implies which: similarity implies congruence or congruence implies similarity. Furthermore, ask for other applications of similarity.

REFERENCES:

Irons, C. and Burnett, J. (1994). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Figure M

Maori Design

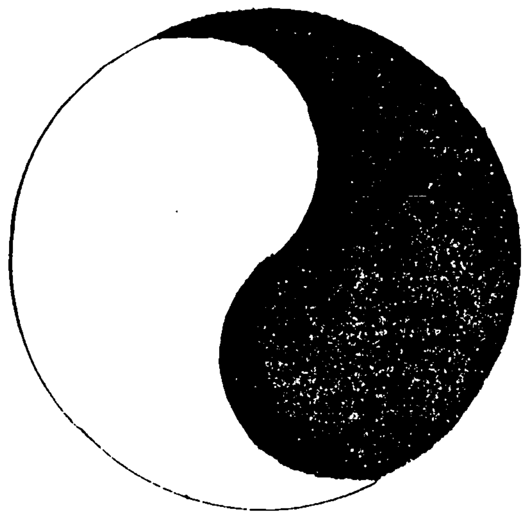


Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

Figure N

Similarity & Congruence Homework Sheet

I. Given the figure below, draw an <u>exact</u> copy of the figure. Additionally, draw a figure that is either <u>twice the original figure</u> or <u>half the size of the original figure</u>.



II. Write a short description on how you made the two figures. What does the figure remind you of?

TOPIC: TRANSFORMATIONS (I)

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, polygons, symmetry, congruence and similarity

AIM: 1) to analyze and define translation, reflection, and rotation, 2) to use the "ALQUERQUE" game board in studying these transformations, 3) to apply these transformations to other patterns

MOTIVATION: Call on people to ask what their favorite game is. After hearing several games, discuss that in the early days before the time of electronic games, some people used board games for recreation.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Show a transparency of the gameboard of Alquerque. Say further that this game is very old, more than a thousand years old. This game, of North African origin, was given the name Alquerque by the Spaniards. In fact, Alfonso the Wise, a Spanish King, wrote the very first book of games in Europe. Next, ask what games are similar to Alquerque -- Chinese checkers, etc. Next, have them focus on the gameboard and ask them to name the figures that they see. After a while, ask whether or not the pattern can be created by just using one "figure." Brainstorm the class on how to do this and then say that the design can be created repeating this "figure." Divide the gameboard into four congruent quadrants and ask how will we create the gameboard using the second quadrant. Show that we can "slide" it to the right and then "slide" the two new figures down, or "slide" it down and then "slide" the two new figures to the right. This process of sliding to form a pattern is called translation. Ask for another translation. Next have them focus on the second quadrant once again and ask how can this quadrant be created by using one single "figure". To do this, divide the second quadrant into two congruent rectangles. Assuming it was divided horizontally, how can we create the second quadrant? This can be done by "flipping" the rectangle to the right. This process of flipping to form a pattern is called reflection. Ask for another reflection. Once again, ask the class to focus their attention to the gameboard and locate the triangle on the top leftmost corner. Ask what kind of a triangle this is (right). Using this triangle, ask how the triangle directly below it can be duplicated. One might answer that it can be reflected along the horizontal leg. Accept this but ask for a method other than a reflection. Point to the vertex of the right angle and ask the student what will happen if without letting go of the vertex, you turn the figure 90 degrees downward? You will get the triangle directly below it! This process of rotating to form a pattern is called rotation. Ask for more rotations.

DRILL: none

MEDIAL SUMMARY: Have them redefine translation, reflection and rotation.

APPLICATIONS AND DRILL: Divide the class into pairs. Ask each pair to create the Alquerque gameboard by using a combination of the transformations. Be prepared to explain the method to the whole class.

FINAL SUMMARY AND CONCLUSION: Read the following to the class: "This game has many other versions around the world. In Sri Lanka, there is a version called PERALIKATUMA. The Zunis in New Mexico plays a version called AWITHLAKNANNAI. The Pueblo Indians likewise have version of this called PICARIA. This game, which most likely was brought to Europe by the North African Moors, is surely one of the pastimes around the world and is still being played today." Afterwards give the class their homework.

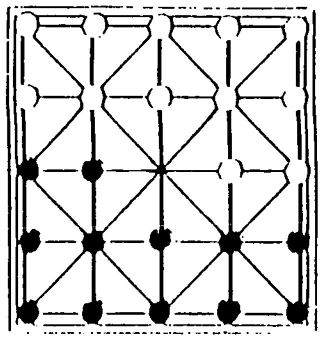
HOMEWORK ASSIGNMENT: Using the uppercase forms of the letters of the English alphabet, have them discover which letters can be created by translation, reflection and rotation.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, overhead projector, transparencies, projection screen, counters

<u>IF TIME:</u> Let them play this game. The rules are as follow: 1) The illustration below shows how to set up the game.

Figure O

Alquerque Gameboard Set-Up



Source: Irons, C. and Burnett, J. (1993). <u>Mathematics from Many Cultures.</u>

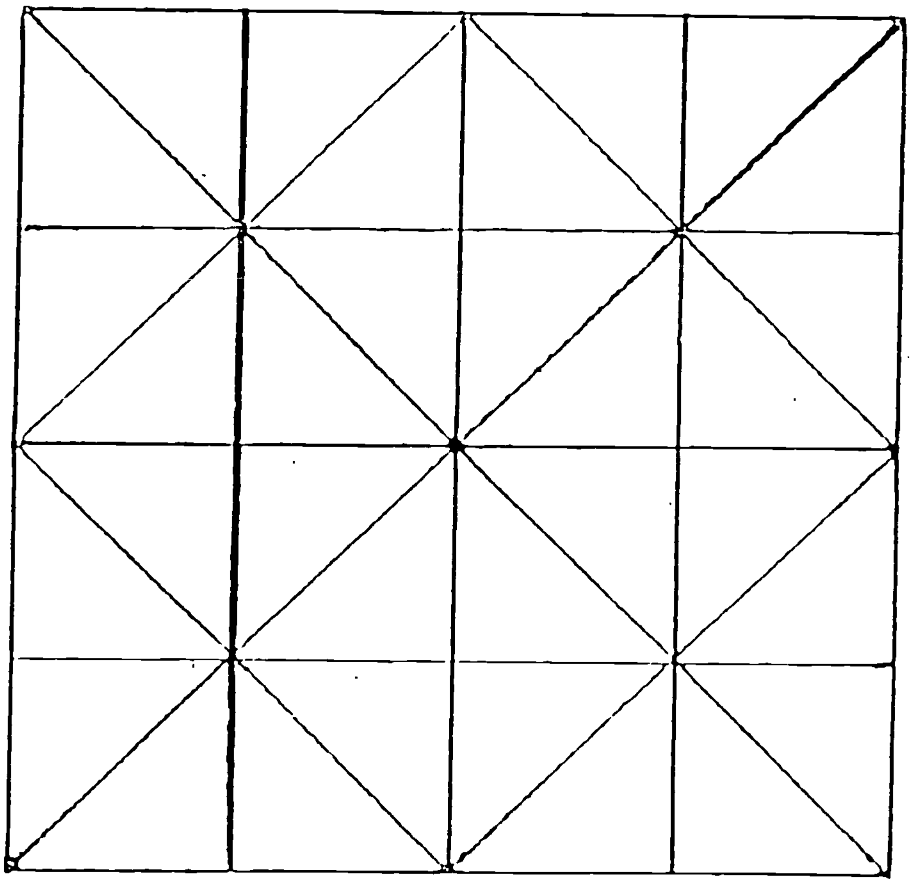
2) Determine who will start. 3) Move your counter in any direction to an empty space. Alternate turns. 4) If your opponent has one counter that is next to an empty space, you must jump over that counter and take it. If you, however, miss that chance, your playmate must take your counter. 5) Whoever takes all the counters of his/her playmate wins.

<u>REFERENCES:</u>

Irons, C. and Burnett, J. (1994). <u>Mathematics from Many Cultures.</u> San Francisco, CA: Mimosa Publications.

Figure P

Alquerque Gameboard



TOPIC: TRANSFORMATIONS (II)

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, polygons, symmetry, transformations

AIM: 1) to apply the transformations into obtaining a pattern, 2) to use different patterns in studying how a certain pattern was obtained by the use of transformations, 3) to draw designs using transformations

MOTIVATION: Ask who "Sherlock Holmes" is. After a few descriptions of who he is, tell them that for today, they will be like him, an investigator.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Group the class into pairs. Hand each group a sheet of paper containing designs from different parts of the world that were created by transformations. Their mission is to analyze these designs and determine how these designs were created. They are to draw the basic figure before the transformations and list the transformations.

DRILL: none

MEDIAL SUMMARY: none

APPLICATIONS AND DRILL: This is a direct application of the transformations. Have them work on as much different designs as possible.

FINAL SUMMARY AND CONCLUSION: Explain that a lot of the intricate designs in the world are made of transformations.

HOMEWORK ASSIGNMENT: Each student will be asked to create a design by transformations. They may color their designs.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper

IF TIME: They may start creating their designs, if time permits.

REFERENCES:

Ascher, M. C. (1991) Ethnomathematics: A Multicultural View of Mathematical Ideas. Pacific Grove, CA: Brooks/Cole Publishing.

Irons, C. and Burnett, J. (1994). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Nelson, D., Joseph, G. G., and Williams, J. (1993). Multicultural Mathematics. Oxford, UK: Oxford University Press.

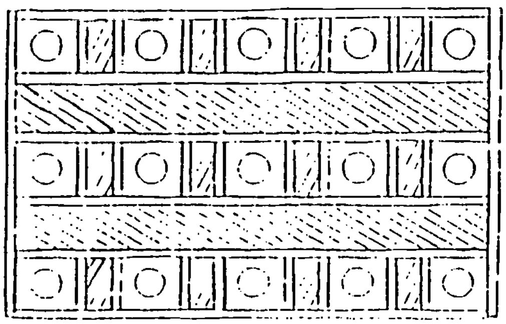
Zaslavsky, C. (1973). Africa Counts. Brooklyn, NY: Lawrence Hill Books.

Figure Q

Transformation Worksheet

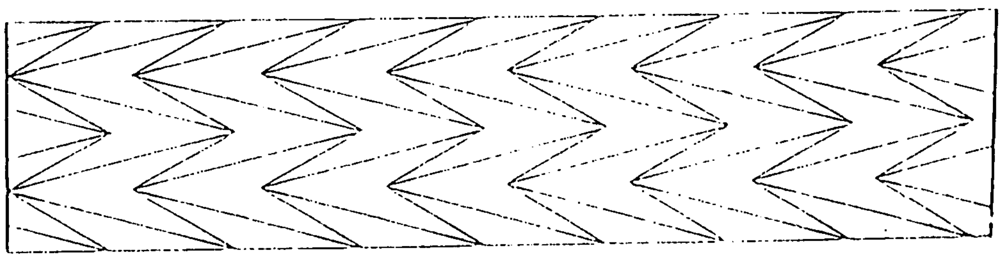
Below are some designs from all over the world. Determine the basic figure from which the design was created. Write the transformations that will create the designs.

1) AUSTRALIAN ABORIGINAL PATTERN



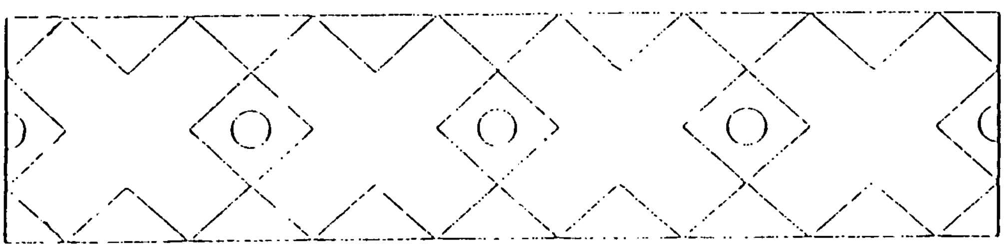
Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

2) MENOMINEE BELT DESIGN (NATIVE AMERICAN)



Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

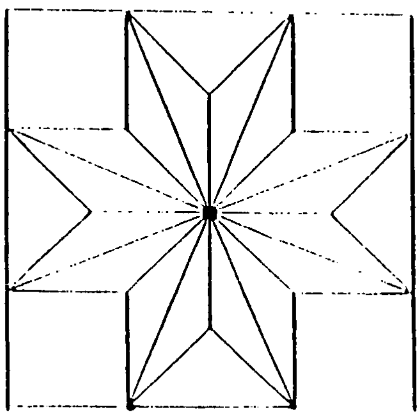
3) MEXICAN HATBAND



Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

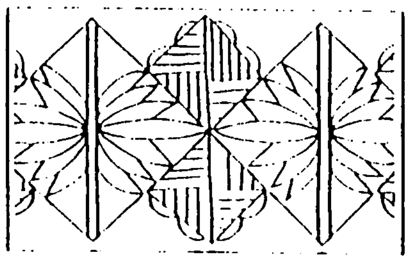
Figure Q (cont'd.)

4) COOK ISLAND DESIGN FROM THE PACIFIC ISLANDS



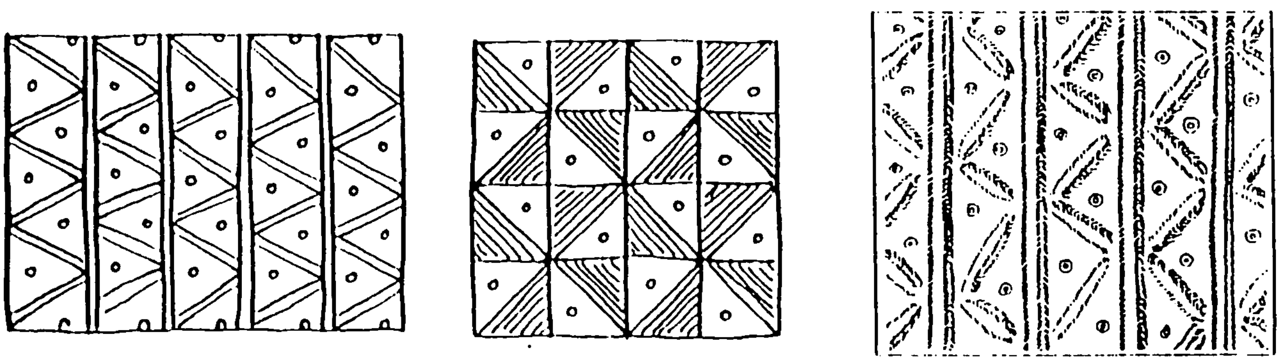
Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

5) SAMOAN PANDANUS PATTERN



Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

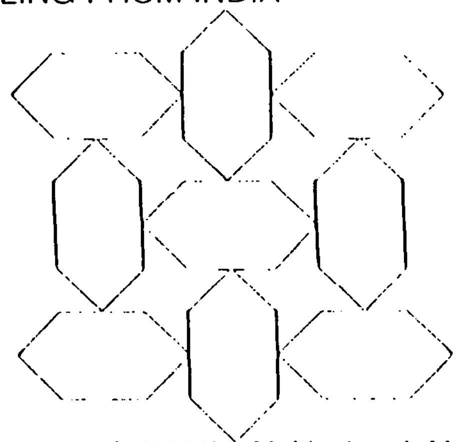
6) YORUBA ADIRE CLOTH DESIGNS OF NIGERIA



Source: Irons, C. and Burnett, J. (1993). Mathematics from Many Cultures.

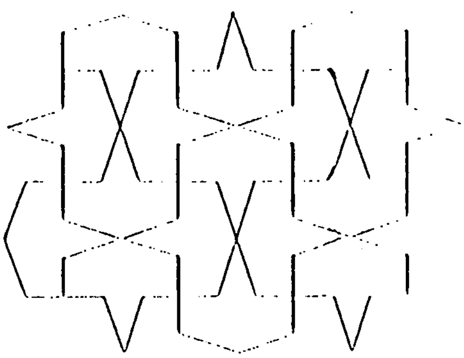
Figure Q (cont'd.)

7) TAJ MAHAL FLOOR TILING FROM INDIA



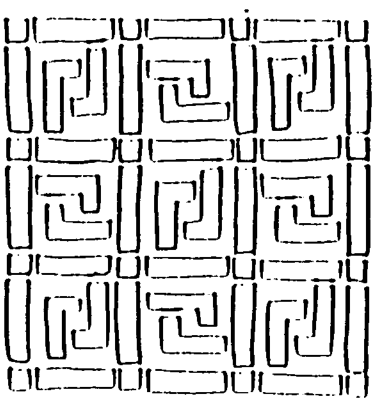
Source: Joseph, G. et al. (1993). Multicultural Mathematics.

8) A DESIGN ON THE TOMB TOWER FROM KHARRAQAN, IRAN



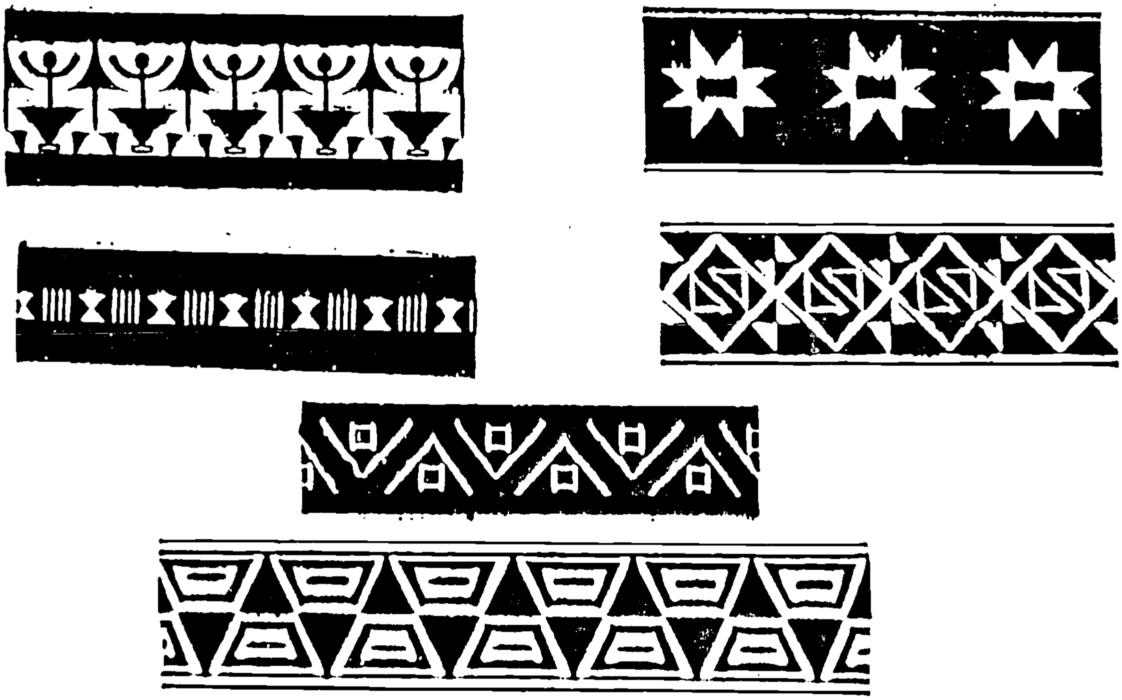
Source: Joseph, G. et al. (1993). Multicultural Mathematics.

9) BAKUBA ART, CONGO



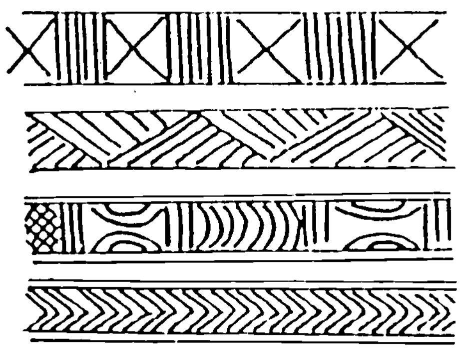
Source: Zaslavsky, C. (1979). Africa Counts.

Figure Q (cont'd.)
10) INCA STRIP PATTERNS, PERU



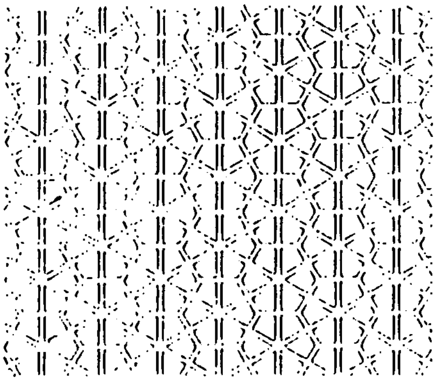
Source: Ascher. M. (1991). Ethnomathematics.

11) BENIN BRONZE PATTERNS, AFRICA



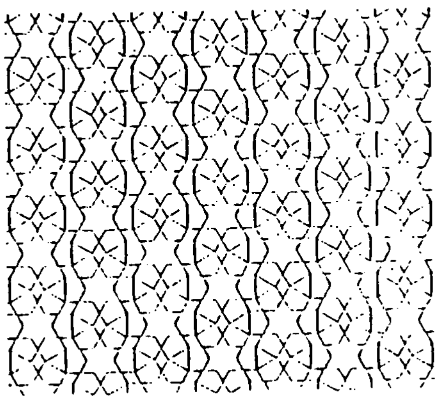
Source: Zaslavsky. C. (1979). Africa Counts.

Figure Q (cont'd.)
12) A CLASSIC CHINESE PATTERN



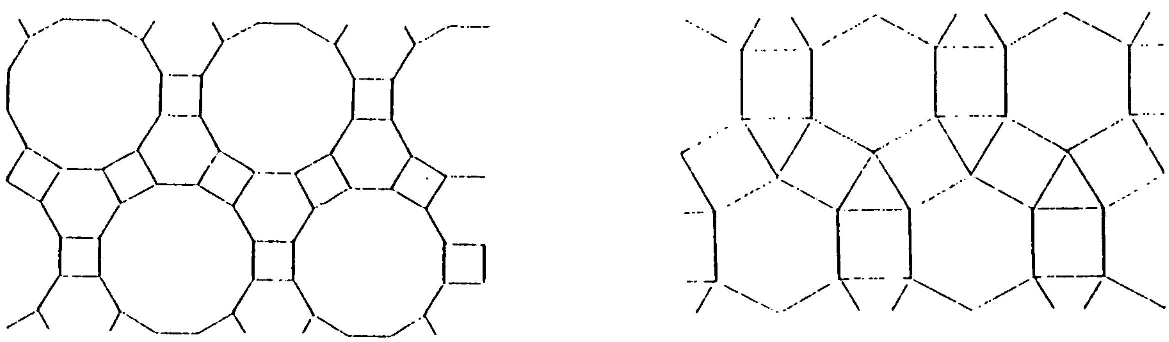
Source: Joseph, G. et al. (1993). Multicultural Mathematics.

13) A TRADITIONAL ARABIC DESIGN



Source: Joseph, G. et al. (1993). Multicultural Mathematics.

14) DESIGNS FROM THE SHIBAM-KAWKABAN, A YEMENESE *MINARET*



Source: Joseph, G. et al. (1993). Multicultural Mathematics.

TOPIC: PRISMS & CYLINDERS

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, polygons, radius, diameter, area, circumference, perimeter

AIM: 1) to use the formulas to solve for the lateral area, surface area, volume of prisms and cylinders, 2) to use the concepts of building structures of different cultures to analyze prisms and cylinders, 3) to apply the lesson by making replicas of the building structures

MOTIVATION: Ask a student to describe his house or apartment. After hearing the description, state that some people live in a different looking house, an example is the Mongolian yurts.

DO-NOW EXERCISE: Write the formulas for the areas of triangle, rectangle, square, parallelogram, trapezoid, and circle.

DEVELOPMENT AND METHODS: Ask someone to describe an apartment building. Ask another student to describe the shape of the base of the building (either a square or a rectangle). Then ask for the area of the this base and call it B. Mention likewise that the base could mean either the one under or the one on top. Next, ask how tall the building is. In real life, what do we call this dimension (height). The figure that is formed is called a PRISM. What then is the amount of space it occupies including the height (volume)? Tell them that there is a formula for this: volume (V) = the area of the base (B) multiplied by the height (H). Ask then for the volume if the bases are square, triangle, trapezoid or parallelogram. Give them time to write down the formulas in their notebooks. Next, what name is given to the figure that has a circular base (cylinder). Have them follow the pattern in finding the volume of the cylinder; $V = \pi r^2 h$. Ask them to focus once again into the shape of an apartment building. If the base is a rectangle, what can they conclude about the sides of the building -- two different rectangles opposite sides are the same, etc. Explain that these side figures are called the lateral sides. Have them guess what to do if they are asked to solve for the lateral area of the figure -- add all the lateral areas. Tell them to write it down in their notebooks. Next, brainstorm the class in determining what surface (total) area means -- (the area of the bases added to the lateral area). Using the same example for the apartment building, ask what the surface area is. Do the same for a prism with square base. Establish the formula for the lateral area of the cylinder. Explain this by asking them to "uncoil" the cylinder. The new figure is a rectangle whose dimensions will be the height of the cylinder and the circumference of the base; hence Lateral Area = $2 \pi r h$. Having established a formula for the lateral area of the cylinder, call on a student to determine the surface area of the cylinder -- lateral area added to the base areas or $SA = (2 \pi r h) + (2 \pi r^2)$.

DRILL: Give a radius and a height and have them solve for the volume, lateral area and surface area of the cylinder.

MEDIAL SUMMARY: none

APPLICATIONS AND DRILL: Continue to ask the class to solve for the volume, lateral area and surface area of the different solids.

FINAL SUMMARY AND CONCLUSION: Make sure that the formulas are written in their notebooks. Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: Ask the class to create a miniature building (prism) and a miniature "yurt" (cylinder). Added directions are: (1) the heights are the same, (2) the base of the building can be any of the learned polygons, (3) design and color them, and (4) solve for the volume, the lateral area and the surface area.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, ruler, notebooks, color pencils or markers, scissors, compass

IF TIME: Engage them into discussing why the varied shapes of the house. Ask what they prefer and justify it.

REFERENCES:

ARAMCO World Magazine. 36 (4) (1985). Leiden, The Netherlands: Aramco Services Company.

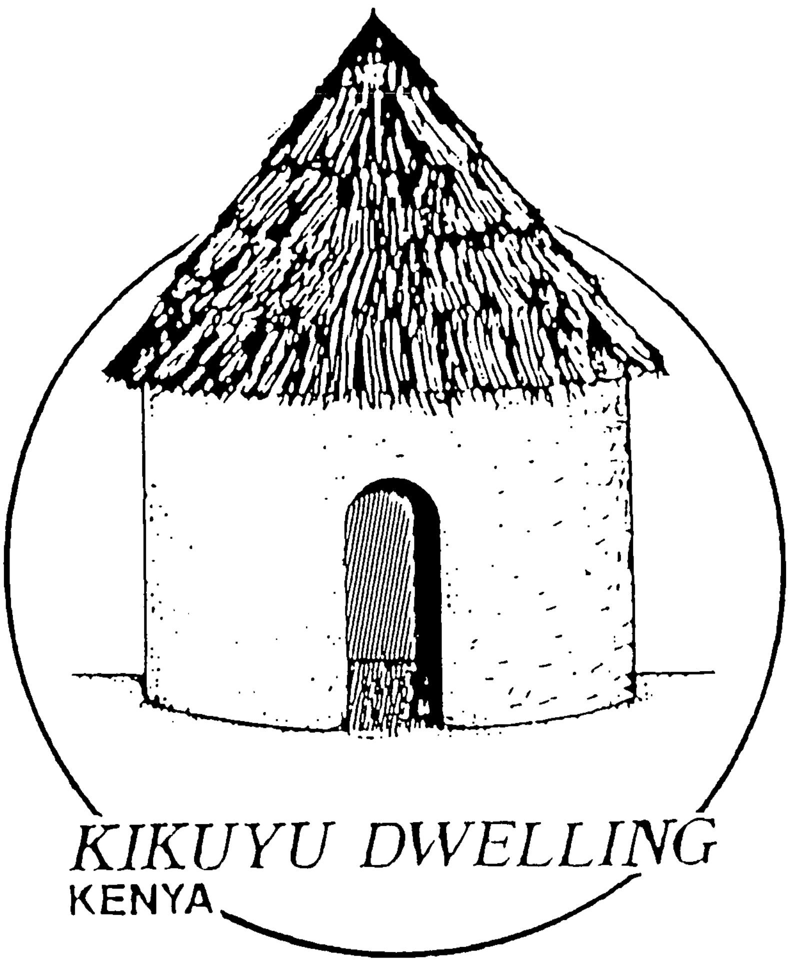
Burton, G. M., et al (1994). Mathematics Plus. Orlando, FL: Harcourt Brace and Company.

Irons, C. and Burnett, J. (1994). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Multicultural Mathematics Posters and Activities (1984). Reston, VA: NCTM Publications.

Figure R

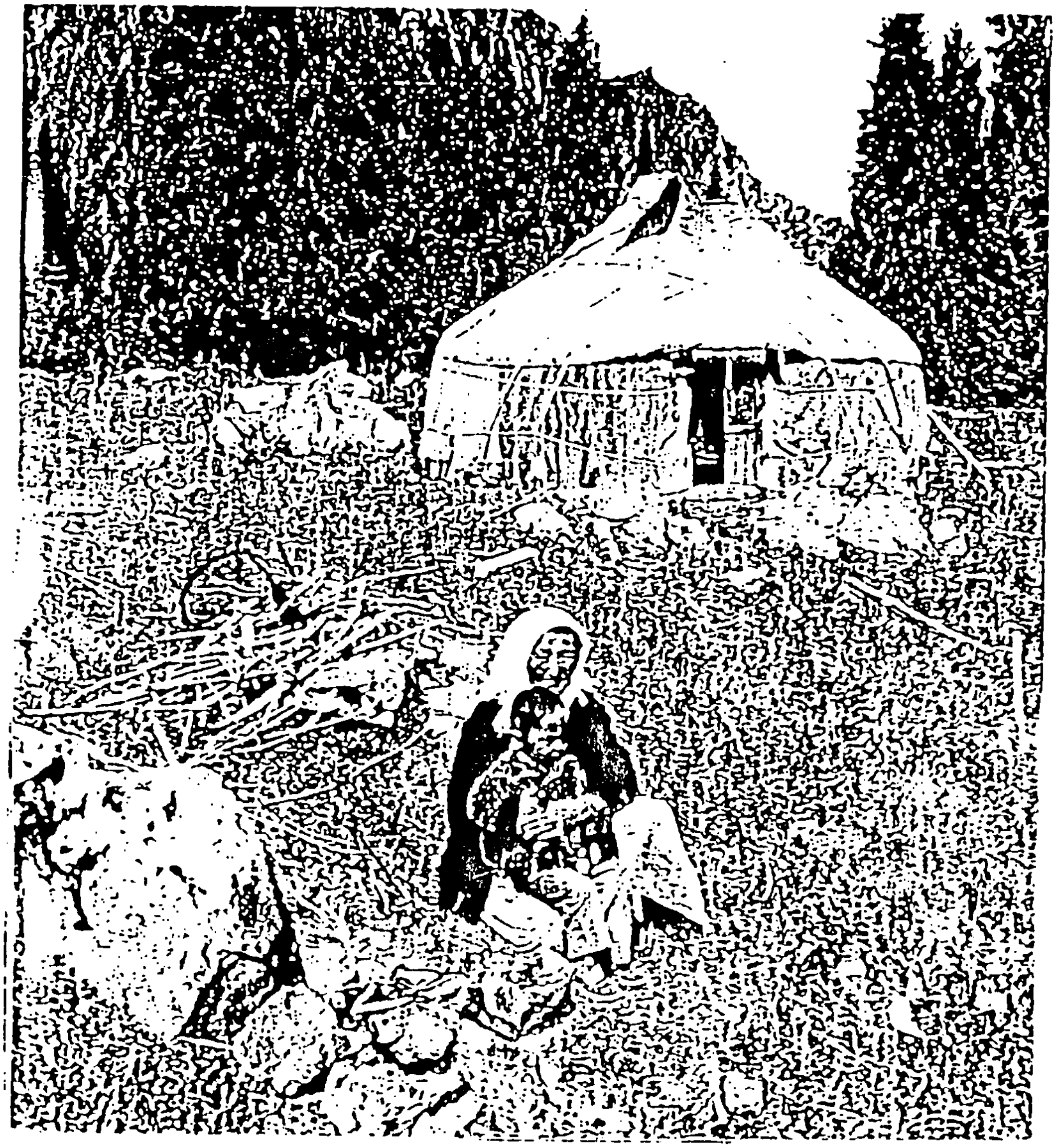
Kikuyu Dwelling



Source: Multicultural Mathematics, Posters and Activities (1984).

Figure S

Mongolian Yurt



Source: ARAMCO World Magazine, (1985).

TOPIC: PYRAMIDS & CONES

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, polygons, radius, diameter, circumference, area, perimeter, prism, cylinder, volume, lateral area, surface area

AIM: 1) to use the formulas to solve for the lateral area, surface area, volume of pyramids and cones, 2) to use the concepts of building structures of different people to analyze pyramids and cones, 3) to apply the lesson by making replicas of the building structures

MOTIVATION: Recall the kinds of building structures that were studied before. Explain that there are other forms of building structures aside from what was already learned.

DO-NOW EXERCISE: Write the formulas for the lateral and surface areas and volumes of cylinders and prisms.

DEVELOPMENT AND METHODS: Ask someone to recall how the formulas for the volume, lateral area and surface areas were established. After listing all the formulas, have them describe a Teepee/Tipi -- circular base, the support all meet in a point above, etc. Ask next what shape this is (cone). Mention that the TEEPEE is not the same as WIGWAM. Explain that WIGWAM is a Chippewa word meaning "dwelling" and is found in the Northeastern part of the United States. Teepees are used in the North American Plains. Next, have them differentiate a cylinder from a cone -- a cylinder has two bases while a cone has one, the top of the cone is a point, etc. What must be done to find the volume of this cone? Explain that the volume of a cone is almost the same as that of the cylinder except for a factor 1/3. Hence, the volume of a cone is $(\pi r^2 h)/3$. What would happen if the base is not a circle but instead a polygon? This figure is called a pyramid. Recall the formula for the volume of a prism. Using the same pattern, what then is the volume of a prism? The volume of a prism has the formula $V = (B h)/3$. Give them time to write the formulas onto their notebooks. Next, ask are the faces of the pyramid -- triangles. How then can the lateral area be solved? Show that for the triangles in the lateral faces of the pyramid, there is a height for this triangle and we call this height the slant height (l). The lateral area of the pyramid is then the sum of the areas of the triangular lateral faces! Show an example at this point. If this is the case, what then is the formula for the surface area of the pyramid -- sum of the lateral area and the area of the base. Show another example. Ask next what is the slant height of the cone -- the distance from the top point to the circle. State the formula for the lateral area of the cone: $\pi r l$, where r is the radius of the circle and l is the slant height of the cone. Ask for a volunteer to state the formula for the surface area of the cone: $(\pi r l) + (\pi r^2)$.

DRILL: More examples will be given to serve as practice.

MEDIAL SUMMARY: none

APPLICATIONS AND DRILL: Continue with the solving. Encourage the students to discuss the formulas amongst themselves and help each other in solving for the answers. The teacher is to go around the classroom to give assistance to those needing it.

FINAL SUMMARY AND CONCLUSION: Make sure that the formulas are written in their notebooks. Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: Ask the class to create a miniature pyramid and a miniature teepee (cone). Added directions are: (1) the heights are the same, (2) the base of the pyramid can be any of the learned regular polygons, (3) design and color them, and (4) the slant heights must be indicated, and 5) solve for the volume, the lateral area and the surface area.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, ruler, notebooks, color pencils or markers, compass, scissors

IF TIME: Have them figure out what will happen if the base of the pyramid is a triangle. Will it really matter if we choose any of the faces as a base? Be prepared to justify the choice.

REFERENCES:

Brundin, J. (1990). The Native People of the Northeast Woodlands, An Educational Resource Publication. New York, NY: Museum of the American Indian - Heye Foundation.

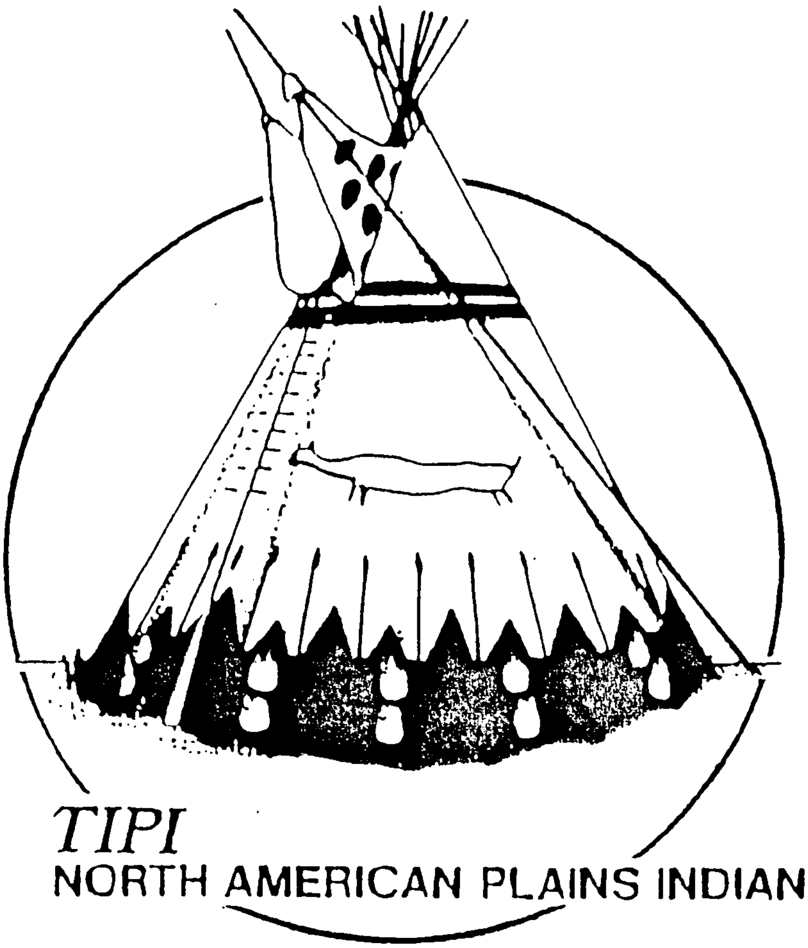
Burton, G. M., et al (1994). Mathematics Plus. Orlando, FL: Harcourt Brace and Company.

Irons, C. and Burnett, J. (1994). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Multicultural Mathematics Posters and Activities (1984). Reston, VA: NCTM Publications.

Figure T

Tipi



Source: Multicultural Mathematics, Posters and Activities. (1984).

TOPIC: SPHERES

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, polygons, radius, diameter, area, volume, surface area, lateral area, circumference, perimeter, prisms, cylinders, cones, pyramids

AIM: 1) to use the formulas to solve for the surface area and volume of spheres, 2) to use the concepts of building structures of different people to analyze spheres, 3) to apply the lesson by making replica of the sphere

MOTIVATION: Recall the different geometric solids that had been studied. Say that there is another form of building structure and this is the igloo and similar shaped house used by some Native American Indians.

DO-NOW EXERCISE: Write the formulas for the volume and lateral and surface areas of the pyramid and cone.

DEVELOPMENT AND METHODS: Ask the class whether they know what an igloo is. Have them describe the igloo. Ask for the shape of the igloo once again. Describe what the base of the igloo is -- circle. Ask what new figure is created if the igloo is reflected along the axis of the base. Ask what is the name for this new figure -- sphere. Ask for some examples of objects that are spherical. If the sphere is observed from the top, what "flat" figure can one observe -- a circle. Use this time to say that the volume of the sphere is given to be $(4 \pi r^3)/3$. Show an example to the class. Furthermore, ask the to imagine "peeling" an orange. The peel is the surface area and to solve for the surface area of the sphere, the formula SA $=(4 \pi r^2)$ is used. Show another example.

DRILL: Give a radius to solve for the surface area and the volume of the sphere.

MEDIAL SUMMARY: Ask some one to reiterate the formulas for the surface area and the volume of the sphere.

APPLICATIONS AND DRILL: Give the class a handout on how to make a dome-shaped house used by the Indians. Mention to the class that this is called the WiGWAM. Explain that dome-shaped houses were used by the MICMAC, EASTERN ABENAKI, and CHIPPEWA. Also, tell the class that some Indians like the KICKAPOO use both the dome-shaped house and the long house. Additionally, give assistance to those requiring it. Encourage the discussion of the formulas.

FINAL SUMMARY AND CONCLUSION: Make sure that the formulas are written in their notebooks. Afterwards, give the class their homework.

HOMEWORK ASSIGNMENT: The homework for this lesson is to make a paper model of a sphere. The instructions are as follow:

A) Draw three circles of the same radius. For the first two circles, draw the radius. For the third circle, divide it into 4 equal quadrants.
B) Cut out the first circle. Cut the radius also. Do the same to the second circle.
C) Slide the first circle into the second circle. Make sure that the centers of both circles meet. Set it aside.
D) Cut out the four quadrants. Attach each quadrant between the first and second circles. Use glue to fasten it well.
E) Solve for the surface area and volume of the sphere you just have created.

SPECIAL EQUIPMENT NEEDED: pen/pencil and paper, ruler, notebooks, color pencils or markers, glue

IF TIME: The students may write about their dome-shaped house.

REFERENCES:

Brundin, J. (1990). The Native People of the Northeast Woodlands, An Educational Resource Publication. New York, NY.: Museum of the American Indian - Heye Foundation.

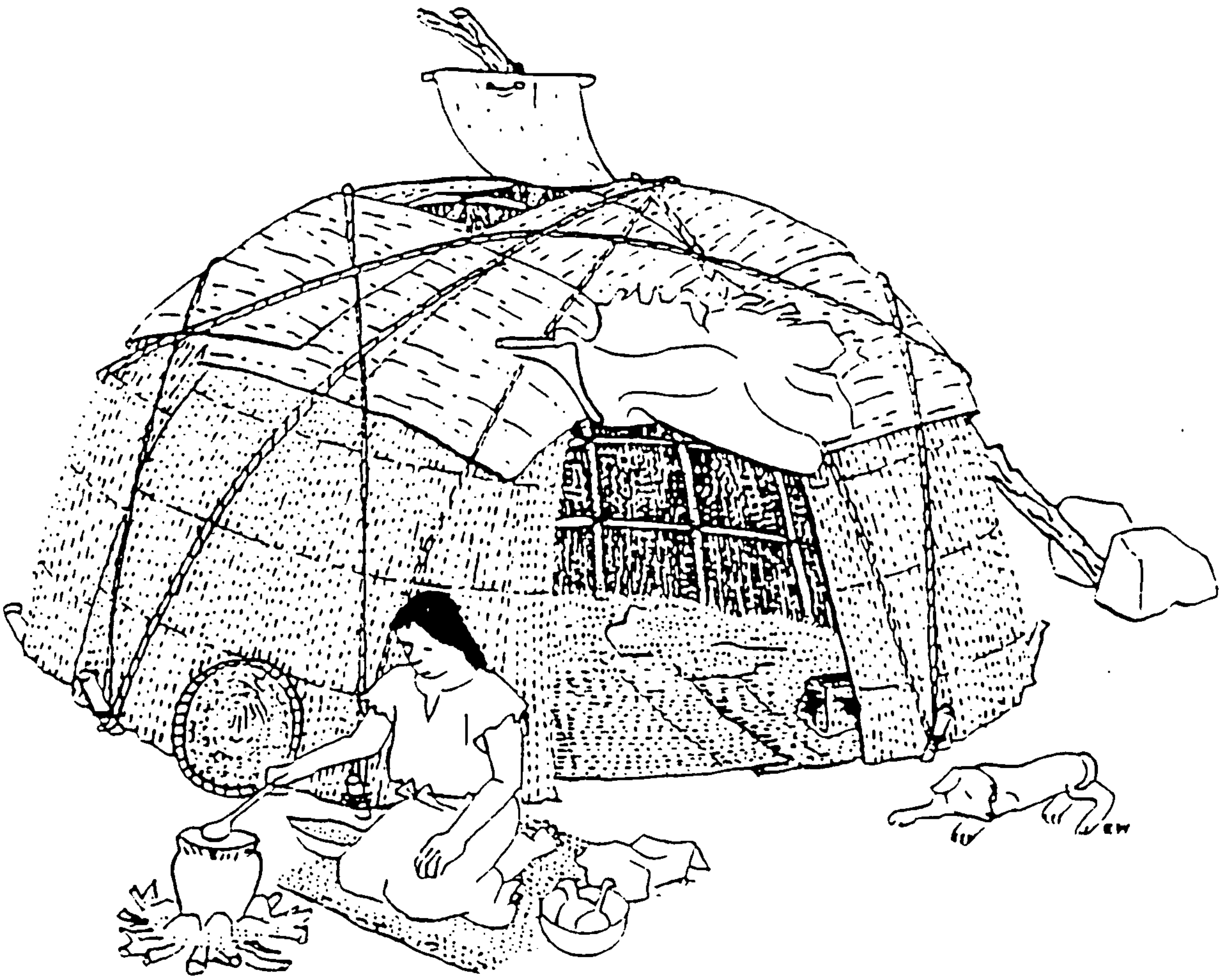
Burton, G. M., et al (1994). Mathematics Plus. Orlando, FL: Harcourt Brace and Company.

Irons, C. and Burnett, J. (1994). Mathematics from Many Cultures. San Francisco, CA: Mimosa Publications.

Multicultural Mathematics Posters and Activities (1984). Reston, VA: NCTM Publications.

Figure U

Dome-Shaped House

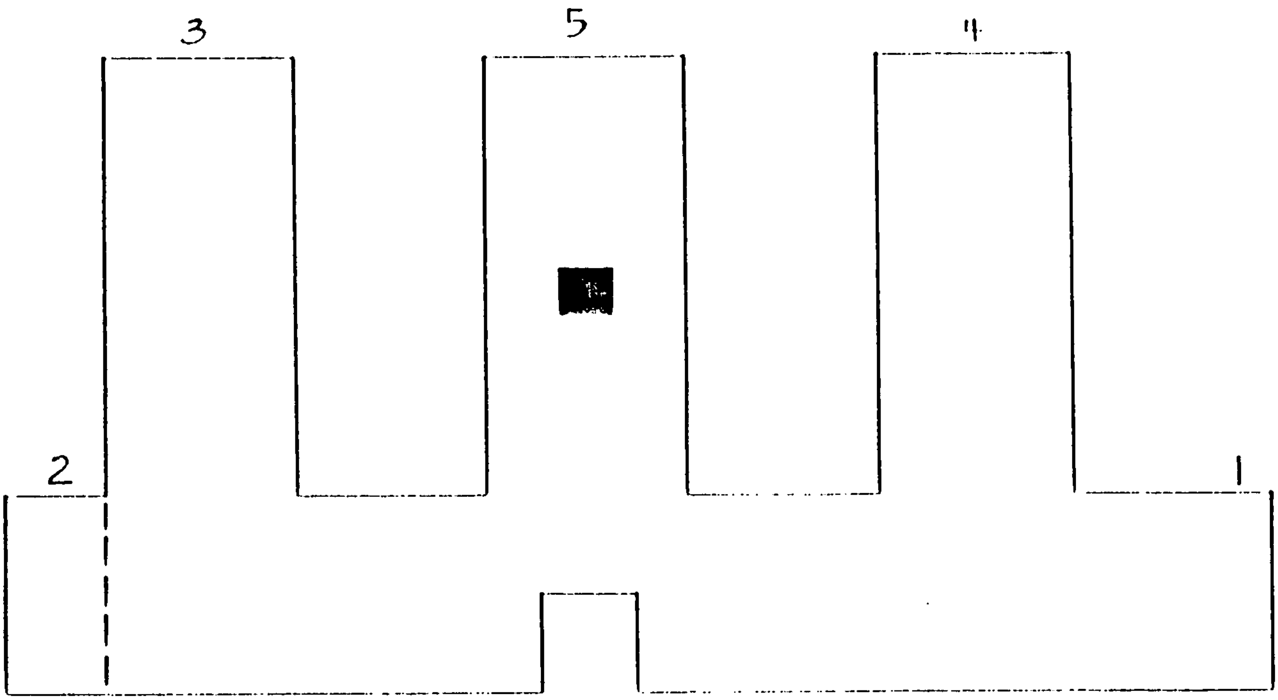


Source: Brundin, J. (1990). The Native People of the Northeast Woodlands.

Figure V

Dome-Shaped House Handout

Connect 1 to 2. Make sure that 1 is on top of 2 and place it until the broken line. Tape or glue them securely. Bend 3 across the top and to the opposite side. Tape or glue it securely. Do the same to 4 and then finally to 5. Bear in mind that 5 is the last one to be bent, since it contains the smokehole. After you have done this, color and design it to look like a real dome-shaped house used by the Native American Indians.



Source: Brundin, J. (1990). The Native People of the Northeast Woodlands.

TOPIC: PROJECTIVE GEOMETRY

PREVIOUSLY LEARNED KNOWLEDGE: points, rays, lines, angles, triangles, radius, diameter, solid geometric figures

AIM: 1) to understand terms like perspective and vanishing point, 2) to name some Renaissance artists who used perspective and vanishing point, 3) to apply the concepts of perspective and vanishing point in drawing a figure

MOTIVATION: Show the transparency of a woodcut of the German artist ALBRECHT DÜRER (1471 - 1528). Ask for some reactions. Explain that he was one of the Renaissance artists that came after the Middle Ages. Furthermore, say that artists like Dürer used a method called PERSPECTIVE.

DO-NOW EXERCISE: none

DEVELOPMENT AND METHODS: Say that perspective is the technique of portraying solid objects on a flat surface so that they appear real. Show the transparency that has the perspective study of Jan Vredeman de Vries (1527 - 1604). Explain that the illustration demonstrates what is called a vanishing point. The vanishing point is the point where parallel lines seem to come together. One example is the railroad tracks meeting in a vanishing point several miles away. Analyze the vanishing point in the perspective study of de Vries. Ask where the vanishing point is located (on the horizon). Show the remaining transparencies of the other Renaissance artists who used perspective.

DRILL: none

MEDIAL SUMMARY: Have them redefine perspective and vanishing point.

APPLICATIONS AND DRILL: The class will be given an opportunity to make use of perspective drawing. Ask them to draw any parallelogram whose base is on the horizon. About 3 inches above the polygon draw a line that is parallel to the base of the parallelogram. Locate the midpoint of the line and mark it. Then, connect all the vertices of the parallelogram to this midpoint. Then, draw a line parallel to the first line. However, make sure that this line is just slightly higher than the parallelogram. To create the 3-D effect, draw lines parallel to the front edges. Erase the rest of the unnecessary lines. Shade the sides and tops of the figure.

FINAL SUMMARY AND CONCLUSION: Say: "There were many paintings in the Middle Ages that were ordered by the Church and the scene was created to be symbolic and little attention was given to correctly depict people and objects. Renaissance artists changed this. They added realism to their work. Some artists and their work include : PIERO DELLA FRANCESCA -- 'The Flagellation' and 'Architectural View of a City;' BOTICELLI -- 'Annunciation of the Virgin;' RAPHAEL -- 'School of Athens;' and GENTILE BELLINI." Afterwards, give the class the homework.

HOMEWORK ASSIGNMENT: Their homework is to write their full names using perspective drawings.

SPECIAL EQUIPMENT NEEDED: pencil and paper, ruler, eraser, overhead projector, transparencies, projection screen

IF TIME: Have them work on figures other than what they have drawn.

REFERENCES:

Burton, G. M., et al (1994). Mathematics Plus. Orlando, FL: Harcourt Brace and Company.

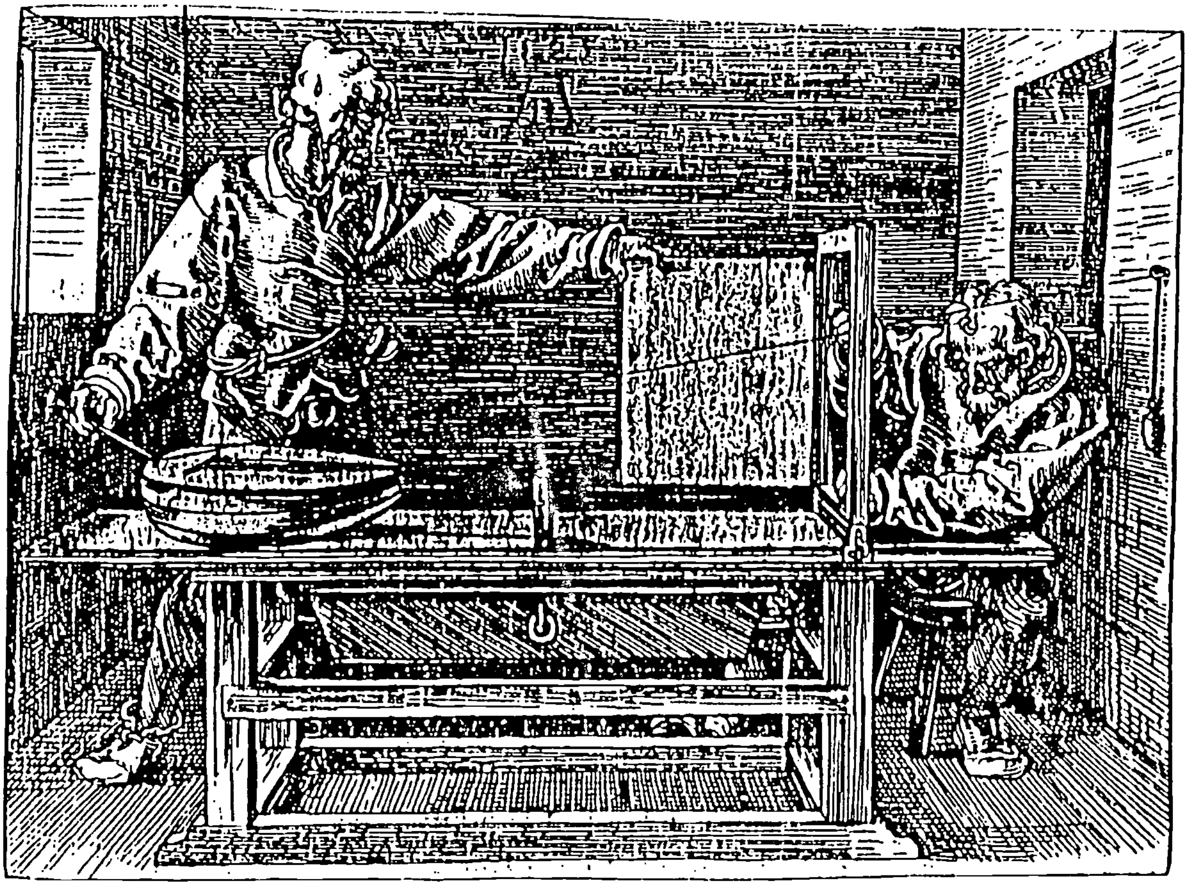
Del Grande, J. J. , Jones, P. T. and Morrow, L. (1982). Mathematics 8. Toronto, Canada: Gage Publishing Limited.

Pappas, T. (1994). The Magic of Mathematics. Discovering the Spell of Mathematics. San Carlos, CA: Wide World Publishing/Tetra.

Serra, M. (1993). Discovering Geometry. An Inductive Approach. Berkeley, CA: Key Curriculum Press.

Figure W

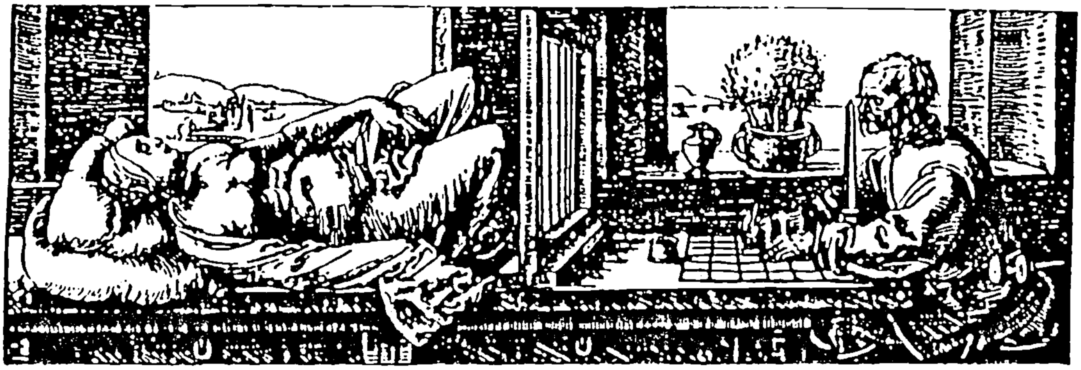
Woodcut of Albrecht Dürer (1)



Source: Serra, M. (1993). Discovering Geometry, An Inductive Approach.

Figure X

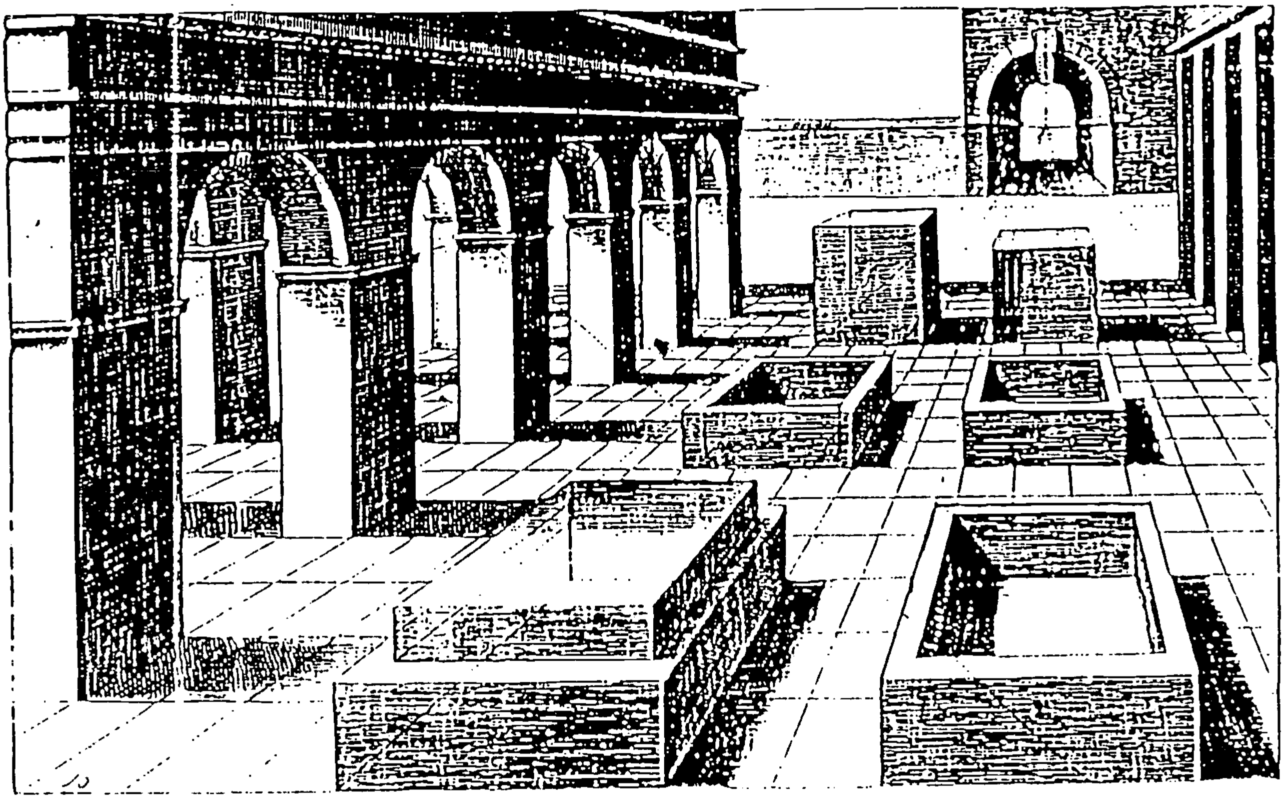
Woodcut of Albrecht Dürer (2)



Source: Pappas, T. (1994). The Magic of Mathematics.

Figure Y

Perspective Study of Jan Vredeman de Vries



Source: Serra, M. (1993). Discovering Geometry, An Inductive Approach.

Appendix B

PRE-ASSESSMENT/POST-ASSESSMENT

PRE-ASSESSMENT(1)

1) $\angle$ A and $\angle$ B are complementary angles and m$\angle$ B = 70°. The supplement of $\angle$ A measures:

a) 180° b) 20° c) 90° d) 160°

2) An angle measuring more than 90° is called:

a) obtuse b) right c) reflex d) acute

3) What is m$\angle$ X if it measures 4/9 of the measure of a straight angle?

a) 40° b) 180° c) 80° d) not given

4) ___________ lines are neither parallel nor intersecting.

a) Perpendicular b) Skew c) Continuous d) Opposite

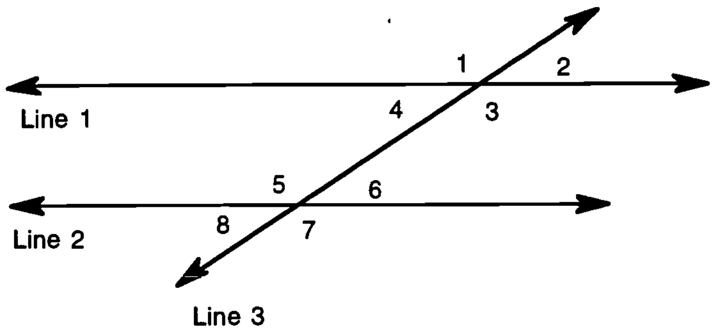
5) Perpendicular lines form a(n) __________ angle.

a) right b) obtuse c) acute d) straight

6) The intersection of two lines is a __________.

a) point b) angle c) plane d) line segment

Use the diagram below to answer questions 7 - 9.



Line 1 is parallel to Line 2.

Line 3 is the transversal.

7) $\angle$ 1 is vertical to ____________________.

8) If m$\angle$ 5 = 85^{o} , what is the measure of $\angle$ 2? _________

9) Name any two alternate interior angles. ____________

10) In ΔABC, m$\angle$ A = 90^{o} and m$\angle$ C = 38^{o}. What is m$\angle$ B ?

a) 52^{o} b) 42^{o} c) 62^{o} d) not given

11) In the above problem, what kind of a triangle is ΔABC?

a) acute b) equilateral c) isosceles d) right

12) ΔXYZ is an isosceles triangle. $\angle$ X and $\angle$ Z are base angles. If $\angle$ Y measures 52^{o}, what is the measure of $\angle$ Z?

a) 90^{o} b) 64^{o} c) 180^{o} d) 128^{o}

13) A square is a rhombus.

a) always b) sometimes c) never d) not given

14) Parallelograms are rectangles.

a) always b) sometimes c) never d) not given

15) **A trapezoid is a parallelogram.**

a) always b) sometimes c) never d) not given

16) **A regular quadrilateral is called a ______.**

a) parallelogram b) square c) trapezoid d) rhombus

17) **A 10-sided polygon is called a ______.**

a) hexagon b) decagon c) nonagon d) octagon

18) **The least number of sides a polygon can have is _____.**

a) one b) two c) three d) four

19) **If the diameter of a circle is 14 m, then the radius is ______.**

a) 3.14 m b) 28 m c) 14 m d) 7 m

20) **The angle formed by 2 adjacent radii measures 1/5 of the circle. What is the measure of this angle?**

a) 72° b) 62° c) 360° d) 20°

21) **The ______ is the intersection of any 2 diameters in a circle.**

a) center b) radius c) arc d) sector

PRE-ASSESSMENT(2)

1) How long is the hypotenuse of a right triangle if one side is 6 and the other side is 8?

a) 36 b) 10 c) 64 d) 100

2 and 3) A bamboo whose upper end, being broken, touches the ground 12 ft. from the stem. The height of the break is 5 ft. How long is the top part of the bamboo? How tall is the bamboo?

a) 25 b) 13 c) 144 d) 169
a) 17 b) 18 c) 25 d) 30

4) What is the perimeter of a regular octagon whose sides measure 9 cm. each?

a) 9 cm b) 9 cm^2 c) 72 cm d) 72 cm^2

5) In a triangle, the base is 16 cm. while the height is 8 cm. Find the area of the triangle.

a) 128 cm b) 128 cm^2 c) 64 cm d) 64 cm^2

6) The parallel sides of a trapezoid measure 20 in and 34 in. The height is 2 ft. What is the area of the trapezoid?

a) 54 in^2 b) 108 in^2 c) 648 in^2 d) 1296 in^2

7 and 8) Given that the diameter is 4.3 cm., find the circumference and the area of this circle. Use 3.14 for π and round the answer to the nearest hundredth.

a) 6.76 cm b) 13.50 cm c) 27.06 cm d) not given
a) 14.51 cm^2 b) 58.06 cm^2 c) 232.24 cm^2 d) not given

9) If the radius of a circle is doubled, then the circumference is ________________.

a) doubled b) tripled c) the same d) halved

PRE-ASSESSMENT(3)

1) The uppercase form of the letter X has ______ line(s) of symmetry.

a) no b) one c) two d) four

2 and 3) A square has ______ line(s) of symmetry and ______ turn symmetry/symmetries.

a) one b) two c) three d) four
a) one b) two c) three d) four

4) Any two squares are similar.

a) always b) sometimes c) never d) not given

5) The ratio of the lengths of 2 rectangles are 3 : 5. For the rectangles to be similar, what has to be the width of the bigger rectangle if the width of the smaller rectangle is 18?

a) 9 b) 30 c) 10/3 d) 1.2

6) Congruent triangles are similar.

a) always b) sometimes c) never d) not given

7) Reflection changes both the size and the shape of a figure.

a) always b) sometimes c) never d) not given

8) Translation preserves both the size and the shape of a figure.

a) always b) sometimes c) never d) not given

9) How many degrees should you rotate an equilateral triangle to get the original figure?

a) 90^0 b) 60^0 c) 30^0 d) 120^0

PRE-ASSESSMENT(4)

1) If the rectangular base of a prism has dimensions 3 cm. and 4 cm., what is the volume if the height is 5 cm.?

a) 60 cm^3 b) 12 cm^3 c) 30 cm^3 d) 24 cm^3

2) Find the lateral area of triangular prism given these conditions: the triangular base has dimensions 7, 3, and 5 and the height is 8.5.

a) 63.75 b) 255.00 c) 127.50 d) not given

3) Find the surface area of a cylinder whose radius is 18 in and whose height is 25 in. Use 3.14 for π and round the answer to the nearest hundredth.

a) 7210248.96 in^2 b) 4860.72 in^2 c) 2939.04 in^2 d) not given

4) If the height of the cone is 12, what is the volume if the radius is 6?

a) 1356.48 b) 452.16 c) 150.72 d) not given

5 and 6) The slant height of a pyramid is 8.5 m. The base is a regular hexagon whose sides measure 7m each. Find the lateral area and the surface area of the pyramid.

a) 168 m^2 b) 59.50 m^2 c) 119 m^2 d) not given

a) 168 m^2 b) 59.50 m^2 c) 119 m^2 d) not given

7) A _______ is half a sphere.

a) hemisphere b) duosphere c) bisphere d) medisphere

8 and 9) If the radius is 2.1 ft., what are the surface area and the volume of this sphere? Use 3.14 for π and round the answer to the nearest hundredth.

a) 55.39 ft^2 b) 26.38 ft^2 c) 13.85 ft^2 d) not given

a) 18.47 ft^3 b) 116.32 ft^3 c) 38.78 ft^3 d) not given

POST-ASSESSMENT(1)

I. Choose the best answer.

1) ∠ A and ∠ B are complementary angles and m∠ B = 70°. The supplement of ∠ A measures:

a) 180° b) 20° c) 90° d) 160°

2) An angle measuring more than 90° is called:

a) obtuse b) right c) reflex d) acute

3) What is m∠ X if it measures 4/9 of the measure of a straight angle?

a) 40° b) 180° c) 80° d) not given

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5) Perpendicular lines form a(n) __________ angle.

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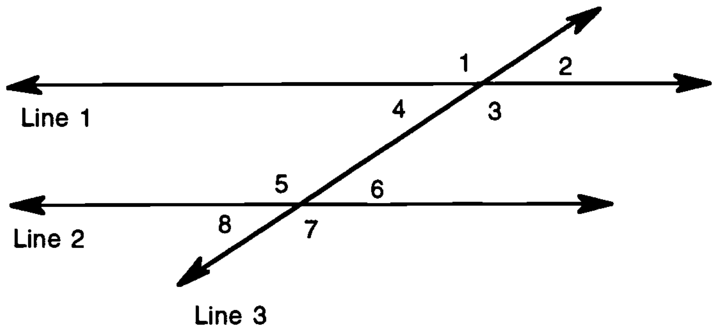

Line 1 is parallel to Line 2.

Line 3 is the transversal.

7) ∠ 1 is vertical to ________________.

8) If m∠ 5 = 85° , what is the measure of ∠ 2?

9) Name any two alternate interior angles. ___________

10) In △ABC, m∠ A = 90° and m∠ C = 38°. What is m∠ B ?

a) 52° b) 42° c) 62° d) not given

11) In the above problem, what kind of a triangle is △ABC?

a) acute b) equilateral c) isosceles d) right

12) △XYZ is an isosceles triangle. ∠ X and ∠ Z are base angles. If ∠ Y measures 52°, what is the measure of ∠ Z?

a) 90° b) 64° c) 180° d) 128°

13) A square is a rhombus.

a) always b) sometimes c) never d) not given

14) Parallelograms are rectangles.

a) always b) sometimes c) never d) not given

15) A trapezoid is a parallelogram.

a) always b) sometimes c) never d) not given

16) A regular quadrilateral is called a ______.

a) parallelogram b) square c) trapezoid d) rhombus

17) A 10-sided polygon is called a _____.

a) hexagon b) decagon c) nonagon d) octagon

18) The least number of sides a polygon can have is _____.

a) one b) two c) three d) four

19) If the diameter of a circle is 14 m, then the radius is _____ .

a) 3.14 m b) 28 m c) 14 m d) 7 m

20) The angle formed by 2 adjacent radii measures 1/5 of the circle. What is the measure of this angle?

a) 72° b) 62° c) 360° d) 20°

21) The _____ is the intersection of any 2 diameters in a circle.

a) center b) radius c) arc d) sector

II. Fill in the blanks.

1) The ________________ and the ancient Egyptians used sundials to tell time.

2) The shadow cast by the ____________________ enables the people to tell the time of day.

3) *Mac* means ____________________.

4) Tartans may have both a name and a ____________________.

5) ____________________ was a great leader of the Zulu tribe.

6) The Zulu tribe is found in ____________________.

7) In Hawaiian, petroglyphs are known as ____________________.

8) ____________________ are primarily used in Hawaiian petroglyphs to illustrate a person's body.

9) The Hopi Indians are found in ____________________.

10) Hopi potters use the ____________________ method to make their pottery.

11) The Tangram is from ____________________.

12) A ____________________ tile was dropped and broke into 7 pieces that eventually became the Tangram.

13 and 14) The ____________________ Medicine Wheel is found in Wyoming and was built by the ____________________ of the Northern Plains.

POST-ASSESSMENT(2)

I. Choose the best answer.

1) How long is the hypotenuse of a right triangle if one side is 6 and the other side is 8?

a) 36 b) 10 c) 64 d) 100

2 and 3) A bamboo whose upper end, being broken, touches the ground 12 ft. from the stem. The height of the break is 5 ft. How long is the top part of the bamboo? How tall is the bamboo?

a) 25 b) 13 c) 144 d) 169
a) 17 b) 18 c) 25 d) 30

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5) In a triangle, the base is 16 cm. while the height is 8 cm. Find the area of the triangle.

a) 128 cm b) 128 cm^2 c) 64 cm d) 64 cm^2

6) The parallel sides of a trapezoid measure 20 in and 34 in. The height is 2 ft. What is the area of the trapezoid?

a) 54 in^2 b) 108 in^2 c) 648 in^2 d) 1296 in^2

7 and 8) Given that the diameter is 4.3 cm., find the circumference and the area of this circle. Use 3.14 for π and round the answer to the nearest hundredth.

a) 6.76 cm b) 13.50 cm c) 27.06 cm d) not given
a) 14.51 cm^2 b) 58.06 cm^2 c) 232.24 cm^2 d) not given

9) If the radius of a circle is doubled, then the circumference is ________________.

a) doubled b) tripled c) the same d) halved

II. Fill in the blanks.

1) The ____________________ were called ***harpedonaptai*** .

2) These rope stretchers reestablished land measurements after the yearly flooding of the ____________________.

3 and 4) The Vedic Sacrificial Altar is found in ________________ and is in the shape of a ____________________.

5) A ____________________ is a design made of concentric circles.

6) ____________________ used the sun and simple geometry concepts to measure the earth's circumference.

POST-ASSESSMENT(3)

I. **Choose the best answer.**

1) The uppercase form of the letter X has ______ line(s) of symmetry.

a) no b) one c) two d) four

2 and 3) A square has ______ line(s) of symmetry and ______ turn symmetry/symmetries.

a) one b) two c) three d) four
a) one b) two c) three d) four

4) Any two squares are similar.

a) always b) sometimes c) never d) not given

5) The ratio of the lengths of 2 rectangles are 3 : 5. For the rectangles to be similar, what has to be the width of the bigger rectangle if the width of the smaller rectangle is 18?

a) 9 b) 30 c) 10/3 d) 1.2

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a) always b) sometimes c) never d) not given

7) Reflection changes both the size and the shape of a figure.

a) always b) sometimes c) never d) not given

8) Translation preserves both the size and the shape of a figure.

a) always b) sometimes c) never d) not given

9) How many degrees should you rotate an equilateral triangle to get the original figure?

a) 90^{o} b) 60^{o} c) 30^{o} d) 120^{o}

II. Fill in the blanks.

1 and 2) The ____________________ Indians who are found in the state of ____________________, weave baskets with symmetric designs.

3) The ____________________ are native people of New Zealand.

4) ____________________ is a symbol for "good and bad," "light and darkness," etc.

5 and 6) The game ____________________ is popular in Spain and was brought to Europe by ____________________ .

POST-ASSESSMENT(4)

I. Choose the best answer.

1) If the rectangular base of a prism has dimensions 3 cm. and 4 cm., what is the volume if the height is 5 cm.?

a) 60 cm^3 b) 12 cm^3 c) 30 cm^3 d) 24 cm^3

2) Find the lateral area of triangular prism given these conditions: the triangular base has dimensions 7, 3, and 5 and the height is 8.5.

a) 63.75 b) 255.00 c) 127.50 d) not given

3) Find the surface area of a cylinder whose radius is 18 in and whose height is 25 in. Use 3.14 for π and round the answer to the nearest hundredth.

a) 7210248.96 in^2 b) 4860.72 in^2 c) 2939.04 in^2 d) not given

4) If the height of the cone is 12, what is the volume if the radius is 6?

a) 1356.48 b) 452.16 c) 150.72 d) not given

5 and 6) The slant height of a pyramid is 8.5 m. The base is a regular hexagon whose sides measure 7m each. Find the lateral area and the surface area of the pyramid.

a) 168 m^2 b) 59.50 m^2 c) 119 m^2 d) not given
a) 168 m^2 b) 59.50 m^2 c) 119 m^2 d) not given

7) A _______ is half a sphere.

a) hemisphere b) duosphere c) bisphere d) medisphere

8 and 9) If the radius is 2.1 ft., what are the surface area and the volume of this sphere? Use 3.14 for π and round the answer to the nearest hundredth.

a) 55.39 ft^2 b) 26.38 ft^2 c) 13.85 ft^2 d) not given
a) 18.47 ft^3 b) 116.32 ft^3 c) 38.78 ft^3 d) not given

II. Fill in the blanks.

1) ____________________ are round houses in Mongolia.

2) The ____________________ house is found in Kenya.

3) A Chippewa word for "dwelling" is ____________________.

4) A ____________________ is made of buffalo hides and is used by the North American Plains Indians.

5) ____________________ Indians, found in the Northeastern part of the United States, used both the dome-shaped house and the rectangular house.

6) A tribe that use a dome-shaped house is ____________________.

7 and 8) After the Middle Ages came the ____________________

and an example of a painter belonging to this age is

________________________________.

Appendix C

STUDENT QUESTIONNAIRES

STUDENT QUESTIONNAIRE(1)

Answer the following questions. For those questions that use a scale of 1 - 5, with 1 being the lowest and 5 being the highest, circle your best answer.

1) I acquired a good understanding of the topic listed below while learning about a direct application of the topic in another culture.

ANGLES	1	2	3	4	5
PARALLEL & PERPENDICULAR LINES	1	2	3	4	5
TRANSVERSALS	1	2	3	4	5
TRIANGLES	1	2	3	4	5
QUADRILATERALS	1	2	3	4	5
POLYGONS	1	2	3	4	5
CIRCLES	1	2	3	4	5

2) I enjoyed the topic listed below while learning about a direct application of the topic in another culture.

ANGLES	1	2	3	4	5
PARALLEL & PERPENDICULAR LINES	1	2	3	4	5
TRANSVERSALS	1	2	3	4	5
TRIANGLES	1	2	3	4	5
QUADRILATERALS	1	2	3	4	5

POLYGONS	1	2	3	4	5
CIRCLES	1	2	3	4	5

3) For each of the topics below, write at least two direct applications or uses of the topic in your own or other people's surroundings.

<u>ANGLES</u>

<u>PARALLEL & PERPENDICULAR LINES</u>

<u>TRANSVERSALS</u>

TRIANGLES

QUADRILATERALS

POLYGONS

CIRCLES

4) Direct applications of the lessons provided me with a better understanding of other cultures.

1 2 3 4 5

5) The lessons involving direct applications provided me with a better appreciation of other cultures.

1 2 3 4 5

6) What topic or lesson did you like most? Why?

7) What topic or lesson did you like least? Why?

8) What was your favorite lesson in this unit? Why?

9) Write any comments or suggestions to improve the approach regarding this unit.

STUDENT QUESTIONNAIRE(2)

Answer the following questions. For those questions that use a scale of 1 - 5, with 1 being the lowest and 5 being the highest, circle your best answer.

1) I acquired a good understanding of the topic listed below while learning about a direct application of the topic in another culture.

RIGHT TRIANGLES	1	2	3	4	5
PERIMETER & AREA OF POLYGONS	1	2	3	4	5
CIRCUMFERENCE & AREA OF CIRCLES	1	2	3	4	5

2) I enjoyed the topic listed below while learning about a direct application of the topic in another culture.

RIGHT TRIANGLES	1	2	3	4	5
PERIMETER & AREA OF POLYGONS	1	2	3	4	5
CIRCUMFERENCE & AREA OF CIRCLES	1	2	3	4	5

3) For each of the topics below, write at least two direct applications or uses of the topic in your own or other people's surroundings.

RIGHT TRIANGLES

____

____

____

____

PERIMETER & AREA OF POLYGONS

CIRCUMFERENCE & AREA OF CIRCLES

4) Direct applications of the lessons provided me with a better understanding of other cultures.

1 2 3 4 5

5) The lessons involving direct applications provided me with a better appreciation of other cultures.

1 2 3 4 5

6) What topic or lesson did you like most? Why?

7) What topic or lesson did you like least? Why?

8) What was your favorite lesson in this unit? Why?

9) Write any comments or suggestions to improve the approach regarding this unit.

STUDENT QUESTIONNAIRE(3)

Answer the following questions. For those questions that use a scale of 1 - 5, with 1 being the lowest and 5 being the highest, circle your best answer.

1) I acquired a good understanding of the topic listed below while learning about a direct application of the topic in another culture.

SYMMETRY	1	2	3	4	5
CONGRUENCE & SIMILARITY	1	2	3	4	5
TRANSFORMATIONS	1	2	3	4	5

2) I enjoyed the topic listed below while learning about a direct application of the topic in another culture.

SYMMETRY	1	2	3	4	5
CONGRUENCE & SIMILARITY	1	2	3	4	5
TRANSFORMATIONS	1	2	3	4	5

3) For each of the topics below, write at least two direct applications or uses of the topic in your own or other people's surroundings.

SYMMETRY

CONGRUENCE & SIMILARITY

TRANSFORMATIONS

4) Direct applications of the lessons provided me with a better understanding of other cultures.

1 2 3 4 5

5) The lessons involving direct applications provided me with a better appreciation of other cultures.

1 2 3 4 5

6) What topic or lesson did you like most? Why?

7) What topic or lesson did you like least? Why?

8) What was your favorite lesson in this unit? Why?

9) Write any comments or suggestions to improve the approach regarding this unit.

STUDENT QUESTIONNAIRE(4)

Answer the following questions. For those questions that use a scale of 1 - 5, with 1 being the lowest and 5 being the highest, circle your best answer.

1) I acquired a good understanding of the topic listed below while learning about a direct application of the topic in another culture.

PRISMS & CYLINDERS	1	2	3	4	5
PYRAMIDS & CONES	1	2	3	4	5
SPHERES	1	2	3	4	5
PROJECTIVE GEOMETRY	1	2	3	4	5

2) I enjoyed the topic listed below while learning about a direct application of the topic in another culture.

PRISMS & CYLINDERS	1	2	3	4	5
PYRAMIDS & CONES	1	2	3	4	5
SPHERES	1	2	3	4	5
PROJECTIVE GEOMETRY	1	2	3	4	5

3) For each of the topics below, write at least two direct applications or uses of the topic in your own or other people's surroundings.

PRISMS & CYLINDERS

PYRAMIDS & CONES

SPHERES

PROJECTIVE GEOMETRY

4) Direct applications of the lessons provided me with a better understanding of other cultures.

1 2 3 4 5

5) The lessons involving direct applications provided me with a better appreciation of other cultures.

1 2 3 4 5

6) **What topic or lesson did you like most? Why?**

7) **What topic or lesson did you like least? Why?**

8) **What was your favorite lesson in this unit? Why?**

9) Write any comments or suggestions to improve the approach regarding this unit.

Appendix D

EVALUATION FORM

EVALUATION FORM

1) NAME OF EVALUATOR: ___________________________

2) IN YOUR OPINION, FOR WHAT GRADE LEVEL(S) ARE THE FOLLOWING LESSONS SUITABLE:

Unit I

ANGLES ___________________________

PARALLEL & PERPENDICULAR LINES ___________________________

TRANSVERSALS ___________________________

TRIANGLES ___________________________

QUADRILATERALS ___________________________

POLYGONS ___________________________

CIRCLES ___________________________

Unit II

RIGHT TRIANGLES ___________________________

PERIMETER & AREA OF POLYGONS ___________________________

CIRCUMFERENCE & AREA OF CIRCLES ___________________________

Unit III

SYMMETRY ________________________________

CONGRUENCE & SIMILARITY ________________________________

TRANSFORMATIONS ________________________________

Unit IV

PRISMS & CYLINDERS ________________________________

PYRAMIDS & CONES ________________________________

SPHERES ________________________________

PROJECTIVE GEOMETRY ________________________________

3) WHAT DO YOU THINK OF THE SEQUENCING OF THE LESSONS?

4) HOW CAN THE LESSONS BE IMPROVED TO ENABLE BETTER USE OF OTHER TEACHERS?

ANGLES ____

____

____

____

PARALLEL &
PERPENDICULAR LINES ______________________________

TRANSVERSALS ______________________________

TRIANGLES ______________________________

QUADRILATERALS ______________________________

POLYGONS ______________________________

CIRCLES ______________________________

RIGHT TRIANGLES

PERIMETER & AREA
OF POLYGONS

CIRCUMFERENCE & AREA
OF CIRCLES

SYMMETRY

CONGRUENCE & SIMILARITY

TRANSFORMATIONS

PRISMS & CYLINDERS

PYRAMIDS & CONES

SPHERES

PROJECTIVE GEOMETRY

5) **WHAT CAN BE DONE TO MAKE THE LESSONS MORE INTERESTING TO THE TEACHERS?**

ANGLES

PARALLEL &
PERPENDICULAR LINES

TRANSVERSALS

TRIANGLES

QUADRILATERALS

POLYGONS

CIRCLES

RIGHT TRIANGLES ______________________________________

____

____

____

PERIMETER & AREA
OF POLYGONS ______________________________________

____

____

____

CIRCUMFERENCE & AREA
OF CIRCLES ________________________________

____

____

____

SYMMETRY ______________________________________

____

____

____

CONGRUENCE & SIMILARITY ______________________________

____

____

____

TRANSFORMATIONS ________________________________

PRISMS & CYLINDERS ________________________________

PYRAMIDS & CONES ________________________________

SPHERES ________________________________

PROJECTIVE GEOMETRY ________________________________

6) WHAT CAN BE DONE TO MAKE THE LESSONS MORE INTERESTING TO THE STUDENTS?

ANGLES ______________________________

____

____

____

PARALLEL &
PERPENDICULAR LINES ______________________________

____

____

____

TRANSVERSALS ______________________________

____

____

____

TRIANGLES ______________________________

____

____

____

QUADRILATERALS ______________________________

____

____

____

POLYGONS ____

____

____

____

CIRCLES ____

____

____

____

RIGHT TRIANGLES ____

____

____

____

PERIMETER & AREA
OF POLYGONS ____

____

____

____

CIRCUMFERENCE & AREA
OF CIRCLES ____

____

____

____

SYMMETRY

CONGRUENCE & SIMILARITY

TRANSFORMATIONS

PRISMS & CYLINDERS

PYRAMIDS & CONES

SPHERES

PROJECTIVE GEOMETRY ________________________________

____

____

____

7) IN YOUR OPINION, HOW DO THESE LESSONS FOSTER/NOT FOSTER THE AWARENESS, APPRECIATION, AND ACKNOWLEDGMENT OF OTHER CULTURES?

ANGLES ____

____

____

____

PARALLEL &
PERPENDICULAR LINES ________________________________

____

____

____

TRANSVERSALS ____________________________________

____

____

____

TRIANGLES ______________________________________

____

____

____

QUADRILATERALS

POLYGONS

CIRCLES

RIGHT TRIANGLES

PERIMETER & AREA
OF POLYGONS

CIRCUMFERENCE & AREA
OF CIRCLES

SYMMETRY

CONGRUENCE & SIMILARITY

TRANSFORMATIONS

PRISMS & CYLINDERS

PYRAMIDS & CONES

SPHERES ____

____

____

____

PROJECTIVE GEOMETRY ______________________________

____

____

____

8) USING A SCALE OF 1 BEING THE LOWEST AND 5 BEING THE HIGHEST, PLEASE RATE THE (I) SUBJECT MATTER, (II) PEDAGOGY, AND (III) CULTURAL AWARENESS OF THE TOPICS.

	I	II	III
ANGLES	_____	_____	_____
PARALLEL & PERPENDICULAR LINES	_____	_____	_____
TRANSVERSALS	_____	_____	_____
TRIANGLES	_____	_____	_____
QUADRILATERALS	_____	_____	_____
POLYGONS	_____	_____	_____
CIRCLES	_____	_____	_____
RIGHT TRIANGLES	_____	_____	_____
PERIMETER & AREA OF POLYGONS	_____	_____	_____
CIRCUMFERENCE & AREA OF CIRCLES	_____	_____	_____
SYMMETRY	_____	_____	_____

CONGRUENCE & SIMILARITY	_____	_____	_____
TRANSFORMATIONS	_____	_____	_____
PRISMS & CYLINDERS	_____	_____	_____
PYRAMIDS & CONES	_____	_____	_____
SPHERES	_____	_____	_____
PROJECTIVE GEOMETRY	_____	_____	_____

9) PLEASE WRITE ANY COMMENTS, SUGGESTIONS, OR RECOMMENDATIONS REGARDING THESE LESSONS PLANS.

SE062594

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I. DOCUMENT IDENTIFICATION:

Title: (1) Mathematics in the Middle Grades: A Multicultural Approach
(2) Geometry

Author(s): FREDERICK L. UY

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Publication Date: 1996

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